1) Give examples of each of the following.
   a) A conditionally convergent series.
   b) A continuous function with domain a bounded interval whose range is unbounded.
   c) A bounded set of rational numbers whose least upper bound is not rational.
   d) A series $\sum_{n=1}^{\infty} a_n$ which diverges but for which the sequence $\{a_n\}_{n=1}^{\infty}$ converges. (The notation $\{a_n\}_{n=1}^{\infty}$ just means $a_1, a_2, a_3, \ldots$)

2) Decide whether and explain briefly why each of the following converges or diverges:
   a) $\sum_{n=1}^{\infty} \frac{\sin n}{2^n}$.
   b) $\sum_{n=1}^{\infty} \frac{3 + \frac{1000 \cos n}{n}}{n^2}$.

3) True or False. Briefly explain your answers. If true, justify; if false, give a counter example.
   a) If $f : \mathbb{R} \to \mathbb{R}$ is uniformly continuous on every closed interval $[-n, n]$ ($n$ a positive integer) then $f$ is uniformly continuous on its entire domain $\mathbb{R}$.
   b) A function $f$ with domain $[0, 1]$ satisfying $f(0) = -1, f(1) = 1$ must have a root $x_0 \in [0, 1]$. (i.e. a value $x_0$ so that $f(x_0) = 0$.)
   d) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n + 1}$ can be rearranged to sum to 2.
   e) If $a_n \leq b_n$ for all $N$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

4) Let $f(x) = \frac{1}{1-x}$ with domain the open interval $(0, 1)$.
   a) Explain why $f(x)$ is continuous on this domain.
   b) Let $\epsilon = 1$ in the definition of uniform continuity. Explain carefully why there is no suitable $\delta$ for this value of $\epsilon$ and consequently $f$ is not uniformly continuous on $(0, 1)$.

5) Explain why
   \[ \lim_{n \to \infty} \frac{1000^n}{n!} = 0. \]