Remember – We don’t expect that everyone will solve every problem, but we do expect that everyone make a serious attempt at every problem and explain what you tried when you can’t solve a problem.

Math 1220, Fall 2017

1) Suppose $|f(x)| \leq x^2$ for all real $x$. Show $f$ is differentiable at $x = 0$.

2) For some value of $a$, define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = \begin{cases} x^2 \cos \frac{1}{x} & x \neq 0 \\ a & x = 0 \end{cases}$. Is there a value of $a$ you can choose making $f(x)$ continuous at for all real $x$? How about differentiable? Can you make the derivative of $f$ continuous for all real $x$?

3) Let $f(x) = a_1 \sin x + a_2 \sin (2x) + \ldots + a_n \sin (nx)$ where $a_1, a_2, \ldots, a_n$ are real numbers and where $n$ is a positive integer. Given that $|f(x)| \leq |\sin x|$ for all real $x$, prove that

$$|a_1 + 2a_2 + \ldots + na_n| \leq 1.$$