1. (12 points) According to the state of California, 33% of all state community college students belong to ethnic minorities. Find the probabilities of the following results in a random sample of 10 California community college students. Do *not* simplify your answers.

   (a) Exactly 2 belong to an ethnic minority.

   (b) Three or fewer belong to an ethnic minority.

   (c) Exactly 5 do not belong to an ethnic minority.

2. (20 points) Instead of the typical sides numbered one through six, an unusual six-sided die has 1 side with the number two, 1 side with the number three, 3 sides with the number four, and 1 side with the number five.

   (a) Draw a histogram for the probability distribution for the possible outcomes of rolling this die.

(b) What is the expected value of a roll of this die?

(c) The die was rolled 5 times giving the outcomes 2, 2, 3, 4, and 4. Draw the histogram for this frequency distribution.

(d) Determine the mean, median, mode(s) and standard deviation of this frequency distribution.

3. (16 points) The chickens at Colonel Thompson's Ranch have a mean weight of 1850 grams, with a standard deviation of 150 grams. The weights of the chickens are closely approximated by a normal curve.

   (a) Find the percent of all chickens having weights in the following ranges.

      (i) More than 1700 grams.

      (ii) Less than 1950 grams.

      (iii) Between 1750 grams and 1900 grams.

   (b) Helen Hen weighs more than approximately 97.5% of the other chickens at Colonel Thompson's Ranch. How much does Helen weigh?

4. (16 points) The National Institute of Health estimates that 10 percent of all Americans 20 years of age or older have diabetes.

   (a) A sample of 100 Americans 20 years of age or older is taken. Use binomial probability only to find *exactly* the probability that

      (i) exactly 16 will have diabetes. Do *not* simplify your answer.

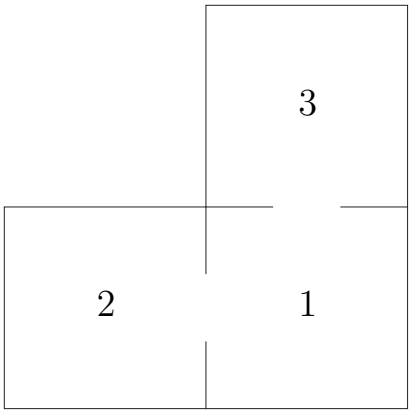
      (ii) more than 16 of the Americans sampled have diabetes. Do *not* simplify your answer.

   (b) Use the normal approximation to the binomial probability of part (a) and the table on the last page to *approximate* the probability that

      (i) more than 16 of the Americans sampled have diabetes. Simplify your answer.

      (ii) exactly 16 of the Americans sampled have diabetes. Simplify your answer.

5. (20 points) A man lives in a house with three rooms and never leaves his house. The only time that the man moves from one room to another is at the start of each hour, at which time the man decides whether or not to move into another room. Half of the time, he stays in the room he is in, and half of the time he chooses an adjoining room at random, and moves into that room. The figure below shows a blueprint of the man's house.



(a) What is the transition matrix of the Markov chain describing the man's position in the house?

(b) Is this Markov chain regular? Justify your answer.

(c) If the man is in room 1, what is the probability that he will be in room 3 two hours from now?

(d) Does this Markov chain have an equilibrium vector? If so, find it. If not, explain why it does not exist.

6. (16 points) A man is playing two slot-machines. The first machine pays off with probability p and, the second with probability q. If he loses, he plays the same machine again; if he wins, he switches to the other machine. This describes a Markov chain with two states, "the man is playing the first machine" and "the man is playing the second machine".

   (a) What is the transition matrix P of this Markov chain?

   (b) Suppose that for his first pull, the man plays the first machine. What is the probability, in terms of p and q, that for his third pull the man plays the second machine?

   (c) Suppose that the second machine never pays off.

      (i) Is this Markov chain regular? Justify your answer.

      (ii) Does this Markov chain have a probability vector V such that $VP = V$? If so, find such a V. If not, explain your answer.

Useful approximate values

$$1/3 \approx 0.33 \quad \text{and} \quad 2/3 \approx 0.67.$$

Area under the standard normal curve

z	Area to left of z
-3	0.0013
-2	0.0228
-1.96	0.0250
-1.83	0.0344
-1.67	0.0475
-1.5	0.0668
-1	0.1587
-0.67	0.2514
-0.33	0.3707