On Teaching

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I have been teaching mathematics, in one form or another, for a long time. Ever since I was in middle school, I have been helping others to solve their problems in math. This early tutoring turned more formal as I progressed through high school and college, where it evolved into grading and running a tutoring lab at Baylor University. All the while, I thought I knew what it took to be a good teacher: present material clearly, accurately and winsomely to the students. However, while those are aspects of a good teacher, what I have come to understand is that there is much more to teaching than presenting material. Through my teaching experiences at Cornell, I have become convinced that most of the best learning occurs when students are discussing ideas. When there is a genuine discussion, not a monologue, students transcend inactive absorption and become actively engaged in the material. When students must argue their point of view, they have to clarify their thinking. They start doing real mathematics; the students begin to see the need for precise definitions, articulate reasoning, and concrete hypotheses. Moreover, when students have an example that they can relate to, they have a hook onto which they can hang their new ideas, making them much easier to remember. The question thus becomes, how can I get students to have this sort of discussion? What I have found is that the key to a good discussion is a good question.

To get the students engaged in this way, a good question has two essential components; the question should relate somehow to the students’ prior experiences, and the answer should not be immediately clear. If the question relates to the students in some way, they are more apt to want to discuss it, and they remember it and the concepts involved in the question. Also, they have an intuitive feel for the answer, which provides a catalyst for their thoughts. If the question’s answer is not immediately clear, in fact, if there is a bit of purposeful ambiguity in the question, the students tend to dig deeper into the topic at hand as they work out among themselves where the ambiguities lie. For example, I have asked my first-semester calculus students whether they could prove that they were once exactly 3 feet tall. Since the question relates to them personally, they take to it quite well. As the discussion progresses from an easy intermediate value theorem proof, they come to realize that the answer relies on whether growth is continuous. Do we grow one discrete cell at a time, or is there some sort of stretching involved? What about when you sleep? Isn’t there a relaxing effect of lying down, and wouldn’t that cause you to stretch out a bit? Thereafter, my students have a much firmer grasp on the intermediate value theorem and its requirement of continuity.

The way in which the questions are asked contributes to how students respond to them. To encourage discussion on such questions, I typically pose the question, allow them to deliberate for a moment, then ask my students to vote for the answer they believe is most correct. The act of voting gets them, in some way, to commit to their answer. This small investment in their answer provides the
stimulus for the discussion. I then ask them to discuss their answer with those around them, and the classroom becomes lively with debate. During this time of discussion, I circulate around the room, clarifying any questions students may have. I am also able to hear what my students are thinking. This, then, allows me to focus on areas that they are struggling with and change my lecture accordingly.

This type of cooperative learning is able to occur because I foster a sense of community and openness in my classroom. My students know that it is safe to ask or say anything they want. I insist that they respect others’ ideas, and I call on many different students during lecture to engage everyone. Further, my lectures are often punctuated with times of other group activities, often involving work on problems that would be too hard for them to tackle by themselves during one class period. For example, I once asked my students to estimate the amount of gold used to cover the dome of the capitol in Denver using differentials. I provided them with the actual data on the capitol dome; I try to tie all of my major examples like this to real situations.

Another aspect of my teaching that has developed at Cornell is my use of technology in my courses. I have used the Blackboard website interface extensively as a way to communicate with my students, posting assignments, worksheets and announcements on the site. I have also used Blackboard to incorporate small pre-class assignments in my recent courses, for which the students are expected to answer a few questions on the chapter we are about to cover. This, I have found, encourages them to read the chapter before coming to class, and, even though they don’t fully understand what they read, they have a passing familiarity with the terms and ideas in the chapter. I am thus enabled to spend time drawing out the major concepts in the material rather than spending time rehashing the textbook to them. Additionally, I have used Maple in my classes to give students a picture of what they are having trouble seeing. In my Game Theory class, for example, I showed my students a plot of an equilibrium point in a simple $2 \times 2$ game. I was able to manipulate the drawing and show them how each player sees the payoffs as well as how the payoffs look from an outside perspective.

Finally, I am starting to explore the benefits of using writing in the mathematics classroom. This semester, in the Game Theory course I am teaching, I have occasionally asked my students to submit a writing assignment based on what we have covered in class. For example, I asked them to write an analysis of a game-theoretic situation they had encountered in their own lives. This forced them to make the connection between what they do every day and what we’re covering in class, as well as requiring them to put their reasoning into words. When students write down their reasoning carefully, they begin to clarify their ideas. Seeing what they have written also allows them to be more reflective than they are often asked to be.

I have grown much as a teacher during my time at Cornell, and I hope to keep improving and innovating as my career progresses. I see my role as a teacher as one of the most significant contributions I can make to the university. I thoroughly enjoy teaching, and I am driven to share my sense of the art of mathematics, the beauty of refined thinking and creativity in the midst of logical boundaries.