Note Taker Checklist Form -*MSRI*

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Talk Title and Workshop assigned to:
Automorphisms of Graph Products / Intro to Geometric Group Theory

Lecturer (Full name): Kim Ruane

Date & Time of Event: 3/28/07 2-3pm

Check List:

( ) Introduce yourself to the lecturer prior to lecture. Tell them that you will be the note taker, and that you will need to make copies of their own notes, if any.

( ) Obtain all presentation materials from lecturer (i.e. Power Point files, etc). This can be done either before the lecture is to begin or after the lecture; please make arrangements with the lecturer as to when you can do this.

( ) Take down all notes from media provided (blackboard, overhead, etc.)

( ) Gather all other lecture materials (i.e. Handouts, etc.)

( ) Scan all materials on PDF scanner in 2nd floor lab (assistance can be provided by Computing Staff) – Scan this sheet first, then materials. In the subject heading, enter the name of the speaker and date of their talk.

Please do **NOT** use **pencil** or colored pens other than black when taking notes as the scanner has a difficult time scanning pencil and other colors.

Please fill in the following after the lecture is done:

1. List 6-12 lecture keywords: CAT(0), boundary, Right-angled Coxeter group, Right-angled Artin group.

2. Please summarize the lecture in 5 or less sentences.
Lectured described of a group action by homeomorphisms on the boundary of X. Examples for right-angled Coxeter groups and right-angled Artin groups

Once the materials on check list above are gathered, please scan ALL materials and send to the Computing Department. Return this form to Larry Patague, Head of Computing (rm 214)

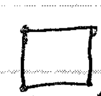
Kim Ruane

let G group act geometrically on X (which is either δ-hyperbolic or it is CAT(0))
neg ↓ non positive ↓

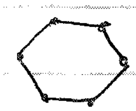
This action extends to an action of G by homeos on $\partial_\infty X$. $\partial_\infty X$ is called the visual bdry of X.

<u>Examples</u>: 1) $G = \mathbb{Z} \oplus \mathbb{Z}$ $\quad X = \mathbb{E}^2 \quad \partial_\infty X \cong S^1$
2) $g \geq 2 \quad G = \pi_1(S_g) \quad X = \mathbb{H}^2 \quad \partial_\infty X \cong S^1$
3) $G = \mathbb{F}_2 \quad X = T \quad \partial_\infty X \cong C$
C = cantor set.

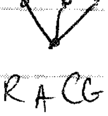
Two points in $\partial_\infty X$ are "close" in visual topology if they travel together for a long time.
4) $G = \mathbb{F}_2 \times \mathbb{Z} \quad X = T \times \mathbb{R} \quad \partial_\infty X \cong \Sigma C$

<u>Replace</u> G by $G_1 = W($  $)$
right angled Coxeter group RACG

right angled Artin group RAAG $A(\bullet\!-\!\bullet)$ (it is like G in ex. 1)

$G_2 = W($  $)$
RACG

$G_3 = W(\cdot\ \cdot\ \cdot)$ or $A(\cdot\ \cdot)$
RACG RAAG

$G_4 = W($  $)$ or $A(\bullet\!-\!\!-\!\!-\!\bullet)$
RACG

they act on same geometries as ex. 1)–4)

Graph Products

Let $\Gamma = (V, E)$ graph

For each $v \in V$, put a group G_v

For each $e \in E$, $e(v,w)$, make G_v commute with G_w

All $G_v = \mathbb{Z}_2$ are RACG ; $G_v = \mathbb{Z}$, RAAG.

co-authors : Adam Piggott & M. Gutierrez,

How does the Aut. gr. of these groups relate to bdry?

In $G_4 = A(a - b - c)$ $\{b\}$ separates the graph Γ

$\Leftrightarrow$ amalgamated prod. splitting

$G_4 = H *_L K$, where $H = \langle a, b \rangle$, $K = \langle b c \rangle$

$L = \langle b \rangle$

Example $a - b - c - d - e$ Γ $W(\Gamma) = W = A *_{\mathbb{Z}_2} B$

RACG $\Rightarrow$ this group has infinitely many ends.

RACG on Γ is one-ended $\Leftrightarrow$ $\nexists$ a complete subgraph that separates.

Theorem : $G \curvearrowright X$ (CAT(0) or δ-hyp), G is one-ended $\Leftrightarrow$ $\partial_\infty X$ connected.

In $A(\bullet - \bullet - \bullet) = H *_L K$ implies that $\partial_\infty X$ is not locally conn.

The $\partial_\infty X$ here is ΣC is connected but not locally connected

theorem : (M-R) If $G = A *_C B$ one-ended group acting on X (CAT(0)) with (i) $\exists\, s \in G$ with $s^n \notin C$ $\forall n \neq 0$, $s C s^{-1} \subseteq C$ (ii) C has to be quasi-convex in X. Then $\partial_\infty X$ is not loc. conn.

Miller / Neumann / Swarup

Construct examples of one-ended G hyperbolic with $|\mathrm{Out}(G)| < \infty$ and having non-trivial JSJ decomp in the sense of Bowditch.

<u>Thm</u> (LeVitt) G 1-ended hyp, then $\mathrm{Out}(G)$ is infinite $\iff$ G splits over a 2-ended sbgr with infinite center.

<u>Claim</u>: If $G = G_1 *_H G_2$ with H 2-ended, ∞ center then $\mathrm{Out}(G)$ is infinite.

<u>Pf</u>: Pick $t \in Z(H)$ (center of H)

$D_t : G \to G$ Dehn twist

$D_t(g_1) = g_1$

$D_2(g_2) = t g_2 t^{-1}$

No power of D_t is inner $\Rightarrow$ D_t is an infinite order elem.

(Separating inters. of links)

<u>Def</u>: Suppose $W(\Gamma)$ is a RACG, Γ has a SIL if $\exists v_i, v_j$ $d(v_i, v_j) = 2$, $\exists R$ comp of $\Gamma \setminus (L_i \cap L_j)$ s.t. $v_i, v_j \notin R$

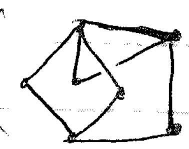
<u>Theorem</u>: TFAE

i) Γ contains a SIL

ii) $\mathrm{Out}(W)$ infinite.

<u>Cor</u> If $\mathrm{Out}(W)$ is infinite, then $\partial_\infty X$ is not loc. conn.

Converse is not true $\to$ see example

$W\Big($  $\Big)$ $\mathrm{Out}(W) =$ finite but $\partial_\infty X$ is non locally conn.