Note Taker Checklist Form - MSRI

Name: Ioana Mihaila

E-mail Address/ Phone #: imihaila@ocu-pomona.edu

Talk Title and Workshop assigned to:
Autonomous Graph Products
Intro to Geometric Group Theory

Lecturer (Full name): Kim Ruane
Date & Time of Event: 8/28/06 2-3 pm

Check List:

( ) Introduce yourself to the lecturer prior to lecture. Tell them that you will be the note taker, and that you will need to make copies of their own notes, if any.

( ) Obtain all presentation materials from lecturer (i.e. Power Point files, etc). This can be done either before the lecture is to begin or after the lecture; please make arrangements with the lecturer as to when you can do this.

( ) Take down all notes from media provided (blackboard, overhead, etc.)

( ) Gather all other lecture materials (i.e. Handouts, etc.)

( ) Scan all materials on PDF scanner in 2nd floor lab (assistance can be provided by Computing Staff) – Scan this sheet first, then materials. In the subject heading, enter the name of the speaker and date of their talk.

Please do NOT use pencil or colored pens other than black when taking notes as the scanner has a difficult time scanning pencil and other colors.

Please fill in the following after the lecture is done:

1. List 6-12 lecture keywords: CAT(0), boundary, right-angled Coxeter, right-angled Artin groups

2. Please summarize the lecture in 5 or less sentences.

Once the materials on check list above are gathered, please scan ALL materials and send to the Computing Department. Return this form to Larry Patague, Head of Computing (rm 214)

For Video Tapings - MSRI 9/2006
let $G$ group act geometrically on $X$ (which is either $\mathbb{S}$-hyperbolic or it is CAT(0))

This action extends to an action of $G$ by homeo on $\partial_\infty X$. $\partial_\infty X$ is called the visual boundary of $X$.

**Examples:**
1. $G = \mathbb{Z} \times \mathbb{Z}$, $X = \mathbb{E}^2$, $\partial_\infty X \cong S^1$
2. $G = \pi_1(S_3)$, $X = \mathbb{H}^2$, $\partial_\infty X \cong S^1$
3. $G = \mathbb{F}_2$, $X = T$, $\partial_\infty X \cong C$
   - $C =$ cantor set.

Two points in $\partial_\infty X$ are "close" in visual topology if they travel together for a long time.

4. $G = \mathbb{F}_2 \times \mathbb{Z}$, $X = T \times \mathbb{R}$, $\partial_\infty X \cong \Sigma C$

Replace $G$ by $G_1 = W(\square)$, right angled Coxeter group $\mathbb{RACG}$

**Right angled Artin group** $\mathbb{A}(\cdots)$ (it is like $G$ in $\mathbb{E}^n$ - 1)

$G_2 = W(\bigstar)$, $\mathbb{RACG}$

$G_3 = W(\bigcirc)$, $\mathbb{RAAG}$

$G_4 = W(\bigtriangleup)$, $\mathbb{RAAG}$

They act on same geometries as ex. 11-4)
Graph Products

Let \( \Gamma = (V, E) \) be a graph.

For each \( v \in V \), put a group \( G_v \).

For each edge \( e \in E \), \( e(v,w) \), make \( G_v \) commute with \( G_w \).

All \( G_v = \mathbb{Z}_2 \) are RACG; \( G_v = \mathbb{Z} \) are RAAG.

Co-authors: Adam Piggott & M. Gutierrez.

How does the Aut. Gr. of these groups relate to \( \Gamma \)?

In \( G_4 = A (\frac{a}{b} \frac{c}{d} \frac{e}{f}) \), \( \frac{bl}{b} \) separates the graph \( \Gamma \).

\[ \Rightarrow \text{ amalgamated prod. splitting } \]

\[ G_4 = H \ast_k K \]

where \( H = \langle a, b \rangle \), \( K = \langle c, d \rangle \)

\[ L = \langle b \rangle \]

Example:

\[ \Gamma \]

\[ w(\Gamma) = W = A \ast \mathbb{Z}_2 B \]

RACG \( \Rightarrow \) this group has infinitely many ends.

RACG on \( \Gamma \) is one-ended \( \iff \) a complete subgraph that separates.

Theorem: \( G \not\cong X \) (CAT(0) or 5-hyp), \( G \) is one-ended \( \Rightarrow \) \( \partial X \) connected.

In \( A (\frac{c}{d} \frac{e}{f}) = H \ast_k K \) implies that

\( \partial_\infty X \) is not locally con.

Theorem: (M-R) If \( G = A \ast \mathbb{Z} \) is one-ended group acting on \( X \) (CAT(0)) with

(i) \( s \in G \) with \( s \neq \theta \), \( s \in C \)

(ii) \( C \) has to be quasi-convex in \( X \). Then \( \partial_\infty X \) is not loc. conn.
Construct examples of one-ended $G$ hyperbolic with $|\text{Out}(G)| < \infty$ and having non-trivial JSJ decompositions in the sense of Bowditch.

Then (LeVitt) $G$ ends hyperbolically, then $\text{Out}(G)$ is infinite $\Rightarrow$ $G$ splits over a 2-ended subgroup with infinite center.

**Claim:** If $G = G_1 \times H \times G_2$ with $H$ 2-ended, 0-center then $\text{Out}(G)$ is infinite.

**Proof:** Pick $t \in \text{Z}(H)$ (center of $H$)

$D_t : G \rightarrow G$ Dehn twist

$D_t(g_1) = g_1$

$D_t(g_2) = t g_2 \cdot t^{-1}$

No power of $D_t$ is inner $\Rightarrow$ $D_t$ is an infinite order element.

**Def.** Suppose $W(F)$ is a RACG, $\Gamma$ has a SIL if

$\exists v_i, v_j \text{ s.t. } d(v_i, v_j) = 2$, $\forall R \in \text{comp of } \Gamma \setminus \{v_i \cup v_j\}$

**Theorem:** TFAE

1) $\Gamma$ contains a SIL
2) $\text{Out}(W)$ infinite.

**Cor.** If $\text{Out}(W)$ is infinite, then $X$ is not locally compact.

Because is not true $\Rightarrow$ see example

$W \left( \begin{array}{c}
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet}
\end{array} \right)$

$\text{Out}(W)$ is finite but $X$ is non-locally compact.