Note Taker Checklist Form -*MSRI*

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Talk Title and Workshop assigned to:
Finiteness properties and Bestvina-Brady Morse theory I / Intro to Geom. Gr. Theory.

Lecturer (Full name): Ian Leary

Date & Time of Event: 8/28/07 10:30-11:20

Check List:

( ) Introduce yourself to the lecturer prior to lecture. Tell them that you will be the note taker, and that you will need to make copies of their own notes, if any.

( ) Obtain all presentation materials from lecturer (i.e. Power Point files, etc). This can be done either before the lecture is to begin or after the lecture; please make arrangements with the lecturer as to when you can do this.

( ) Take down all notes from media provided (blackboard, overhead, etc.)

( ) Gather all other lecture materials (i.e. Handouts, etc.)

( ) Scan all materials on PDF scanner in 2nd floor lab (assistance can be provided by Computing Staff) – Scan this sheet first, then materials. In the subject heading, enter the name of the speaker and date of their talk.

Please do **NOT** use **pencil** or colored pens other than black when taking notes as the scanner has a difficult time scanning pencil and other colors.

Please fill in the following after the lecture is done:

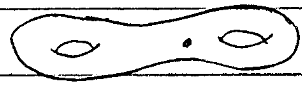
1. List 6-12 lecture keywords: Finiteness conditions, homological algebra, FP, FL, ~~class~~ Eilenberg-Mac Lane space, classifying space.

2. Please summarize the lecture in 5 or less sentences.
The finiteness conditions F, F_∞, F_n, FP, FP_∞, FP_n and some others were defined, and the implications between them discussed. The Bieri-Eckmann homological criterion for FP was proved.

Once the materials on check list above are gathered, please scan ALL materials and send to the Computing Department. Return this form to Larry Patague, Head of Computing (rm 214)

Ian Leary

$K(G,1)$ is a CW-complex with $\pi_1 = G$, connected and its univ. covering space is contractible.



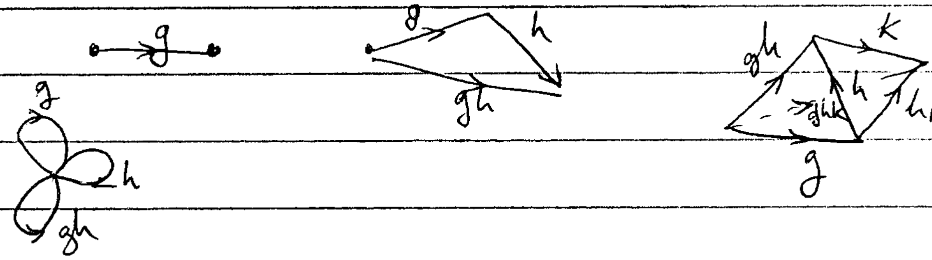
We can always take a finite 0-skeleton, a 1-skeleton if G is f.g., a 2-skeleton if it is f. presented.

G is type F if it has a finite $K(G,1)$

type F_n if $\exists\, K(G,1)$ with finite n-skeleton.

F_∞ $\exists\, K(G,1)$ with all skeleton finite, that is F_n, $\forall n \in \mathbb{N}$.

The bar construction: n-cells G^n



$$H^*(C_n;\mathbb{Z}) = \begin{cases} \mathbb{Z}/n & \text{if } * \text{ even}, >0 \\ \mathbb{Z} & \text{if } * = 0 \\ 0 & \text{if } * \text{ is odd} \end{cases}$$

In particular, any non-trivial finite group is F_∞ but not F. Type F also implies torsion free.

Take the universal cover of a $K(G,1)$. G is acting freely on it. This is a G-CW complex E, which is contractible.

The cellular chain complex $C_*(E)$ is a chain complex of free $\mathbb{Z}G$-modules, which is exact except it needs a $\mathbb{Z}$ added in degree -1.

Hence it is a free resolution of $\mathbb{Z}$ as a $\mathbb{Z}G$ module.

Types: FL – $\exists$ finite free resolution of $\mathbb{Z}$ by fin. gen. frees

FP – $\exists$ finite res of $\mathbb{Z}$ by f.g. projectives

FP_∞ – $\exists$ res of $\mathbb{Z}$ by f.g. projectives

FP_n – $\exists$ res of $\mathbb{Z}$ in which first n-terms are f.g. projs.

Same for FL_∞, FL_n

FH – $\exists$ a free G-CW complex which is acyclic & finitely many orbits of cells.

FH_∞

FH_n

Let M be a module over a ring R (could be $\mathbb{Z}$)

$$\cdots P_2 \to P_1 \to P_0 \to M \to 0$$
$$0 \to \oplus \xrightarrow{id} \oplus$$
$$Q_0 \quad\quad Q_0 \to 0$$
$$0 \to Q_1 \to \oplus$$
$$Q_1 \to 0$$

Pick Q_0 s.t. $P_0 \oplus Q_0$ ~~is~~ free

$$FL_n \equiv FP_n$$
$$FL_\infty \equiv FP_\infty$$

Difference (if any) between FP & FL is measured by $K_0(R)$ (for R-modules)

Start with the monoid of isom. classes of f.g. projectives, make an abelian group by formally adding inverses $\mathbb{N} \rightsquigarrow \mathbb{Z}$.

Example: $R = \mathbb{Z}/6$ $\quad \mathbb{Z}/2$ & $\mathbb{Z}/3$ are projective but not free. The monoid of projectives is $\mathbb{N}^2$ the group $K_0(R)$ is $\mathbb{Z}^2$.

If M is FP, with FP resolution

$$0 \to P_n \to \cdots \to P_0 \to M \to 0$$

then M is FL iff $\sum (-1)^i [P_i] = [R^{\oplus k}]$ in $K_0(R)$ for some k.

Group homology / cohomology detects FP

TFAE: (1) A module M is FP_n over R

(2) $\forall$ exact limit (for ex. a direct product) N_*, the map $\mathrm{Tor}_k^R(\lim N_\alpha, M) \to \lim \mathrm{Tor}_k^R(N_\alpha, M)$ is iso for $k<n$ and epi for $k=n$

(3) For any indexing set I, the map $\mathrm{Tor}_k^R(\prod_I R, M) \to \prod_I \mathrm{Tor}_k^R(R, M)$ is iso for $k<n$ & epi for $k=n$.

~~(4) For any exact co~~

(1)$\Rightarrow$(2) easy

(2)$\Rightarrow$(3) trivial, just a special case.

(3)$\Rightarrow$(1) ~~take $I=M$~~ by induction.

$$K \rightarrowtail F \twoheadrightarrow M$$

K is FP_{n-1} $\Leftrightarrow$ M is FP_n

Case $n=0$ take $I=M$. Then

$$\left(\prod_{m\in M} R\right)\otimes M \twoheadrightarrow \prod_{m\in M} M$$

Have an element of RHS $\underline{m}$ in M_m

$$\sum \underline{r}^{(i)} \otimes m_i \longrightarrow \underline{m}$$

these finitely many m_i's generate.