Note Taker Checklist Form -*MSRI*

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Talk Title and Workshop assigned to:
Moduli space & Thurston's classification /
Intro to Geometric Group Theory

Lecturer (Full name): Benson Farb

Date & Time of Event: 8/29/07, 10:30 - 11:30

Check List:

( ) Introduce yourself to the lecturer prior to lecture. Tell them that you will be the note taker, and that you will need to make copies of their own notes, if any.

( ) Obtain all presentation materials from lecturer (i.e. Power Point files, etc). This can be done either before the lecture is to begin or after the lecture; please make arrangements with the lecturer as to when you can do this.

( ) Take down all notes from media provided (blackboard, overhead, etc.)

( ) Gather all other lecture materials (i.e. Handouts, etc.)

( ) Scan all materials on PDF scanner in 2nd floor lab (assistance can be provided by Computing Staff) – Scan this sheet first, then materials. In the subject heading, enter the name of the speaker and date of their talk.

Please do **NOT** use **pencil** or colored pens other than black when taking notes as the scanner has a difficult time scanning pencil and other colors.

Please fill in the following after the lecture is done:

1. List 6-12 lecture keywords: Teichmüller Space, Moduli Space, Teich metric, Nielsen-Thurston Classification, Pseudo-Anosov homeomorphism

2. Please summarize the lecture in 5 or less sentences.
Description of an arbitrary elem of Mod_g. Definition and basic prop. of Teichmüller Space. Statement of Thurston (Nielsen) classification theorem and idea of proof.

Once the materials on check list above are gathered, please scan ALL materials and send to the Computing Department. Return this form to Larry Patague, Head of Computing (rm 214)

Benson Farb II

Analogy

Vector space	Surface
$\mathbb{H}^n$	Teich (S)
lattice Γ	Mod (S)
$\Gamma \backslash \mathbb{H}^n$ complete finite vol hyp orbifold	$\mathcal{M}(S)$ = moduli space
$\varphi \in \Gamma$ { finite order, parabolic, hyperbolic	$\varphi \in$ Mod S { finite order, reducible, pseudo-Anosov
unique axis in $\mathbb{H}^n$ for hyp φ	unique axis in Teich (S) for φ pseudo
Jordan form	Thurston normal form
eigenspaces, foliations by parallel copies	invariant subsurfaces, invar foliations
eigen-values	dilatations on pseudo or subsurf
$\partial\mathbb{H}^n$	$\mathcal{PMF}$, Thurston bdry

Goals: Σ_g = closed genus g surf.
$\mathrm{Mod}_g = \mathrm{Mod}(\Sigma_g)$

(1) What does an arbitrary elem $\varphi \in \mathrm{Mod}_g$ look like? → think eigen-values, Jordan form of a matrix

(2) Describe $\mathcal{M}_g$ = moduli space
= {hyp metrics on Σ_g}/~
= {conformal classes of Riem. metrics}
= {complex structures}
= {alg. curves}
⋮

The torus

Recall $\mathrm{Mod}(T^2) \cong SL_2\mathbb{Z}$

$$\varphi_A \longleftrightarrow A$$

Prop: Every $A \in SL_2\mathbb{Z}$ is conj to one of:

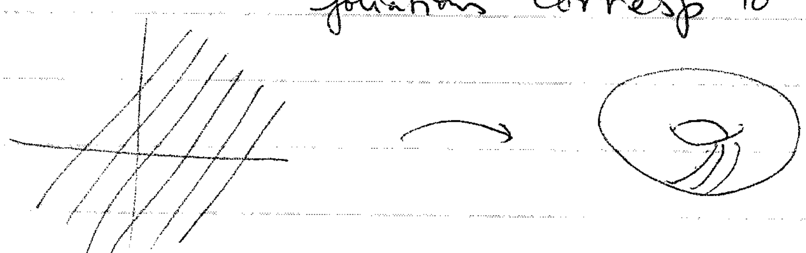
(1) finite order

(2) $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ some $n \in \mathbb{Z}$

(3) $\begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix}$, $\lambda > 1$ $\rightsquigarrow$ case when $|\mathrm{Tr} A| > 2$

Structure of case (3)

1. There are 2 invariant irrational foliations corresp to eigenspaces of A.



2. area - preserving
3. periodic pts for φ_A are dense in the torus
4. $\forall$ scc $\beta, \gamma \subset T^2$, $i(\varphi_A^n(\gamma), \beta) \approx \lambda^n$.
5. entropy.

Proofs of Prop $\to$ proof #1 Jordan form

$\to$ proofs #2, 3 Look at action on $\mathbb{H}^2$

#2 Look at minset(A)

#3 A acts on $(\mathbb{H}^2 \cup \partial\mathbb{H}^2) \cong \overline{D}^2$. Apply Brower's fixed pt.

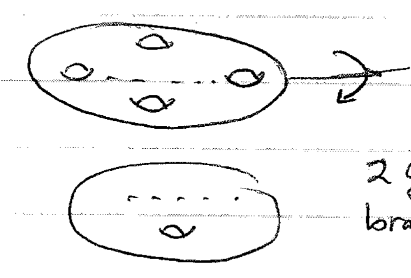
An example in higher genus.

Σ_g

2 fold branch cover

$\downarrow$

T^2



$2g-2$ branch pts

Let A be a hyp matrix ($|\mathrm{Tr} A| > 2$)

$\varphi_A \in \mathrm{Homeo}^+(T^2)$

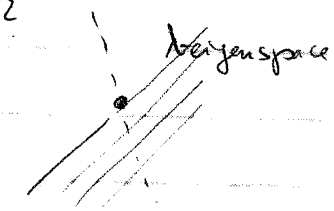
Check that φ_A^m lifts for some m to a homeo ψ of

What does Ψ look like?

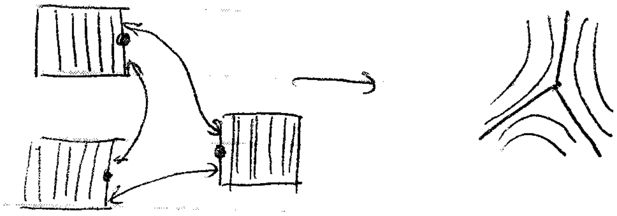
$\exists$ piecewise flat metric on Σ_g s.t. Ψ is at least locally affine. If $z = x+iy$ $\varphi(z) = \lambda x + i\lambda^{-1} y$.

Near the branch pts:

Σ_g 

T^2 

eigenspace



Ψ is pseudo-Anosov.

Ψ has great properties (see 1-5 above)

Nielsen - Thurston classification

$g \geq 1$

Every $\varphi \in Homeo^+(\Sigma_g)$ is homotopic to some $\Psi \in Homeo^+(\Sigma_g)$ with:

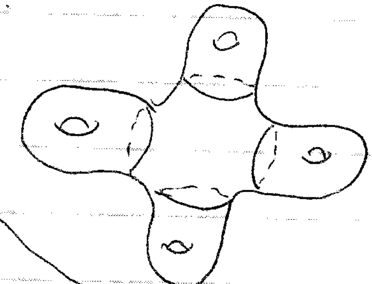
(1) Ψ is finite order

(2) Ψ is reducible, i.e $\exists$ scc $\gamma \subset \Sigma_g$, $\exists$ $m \geq 1$ with $\Psi^m(\gamma) \sim \gamma$

(3) Ψ is pseudo-Anosov

Cor: Norm. form Thurston.

$\varphi \in Mod_g$ is



Applications:

- classif of surface bundles over a circle $\Sigma_g \to M^3 \downarrow S^1$
- group theory of mods

Teich Space Basics

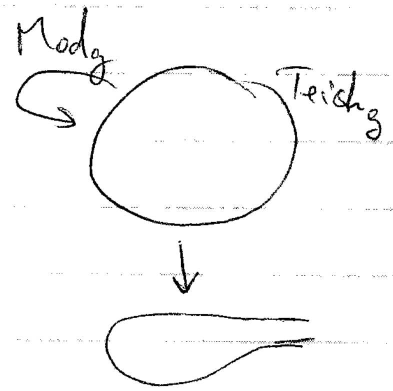
$$\mathrm{Teich}_g = \{ (X, f) \mid X \text{ hyperb surface}, f : \Sigma_g \xrightarrow{\approx} X \}/\sim \qquad g \geq 1$$

$$\mathrm{Teich}_1 = \mathbb{H}^2$$

$$\mathbb{R}^2 / \mathrm{Span}\{ \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix} \} \qquad \mathbb{R}^2 / \mathrm{Span}\{ \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix} \}$$

different in Teich_1, even though they are isom. tori.

- Mod_g acts on Teich_g via $\varphi.(X, f) := (X, f \circ \varphi^{-1})$

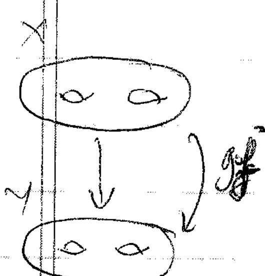


$$\mathcal{M}_g := \mathrm{Mod}_g \backslash \mathrm{Teich}_g$$

$$\pi_1^{orb}(\mathcal{M}_g) = \mathrm{Mod}_g.$$

Define $d_{\mathrm{Teich}_g}((X, f), (Y, g)) :=$

$$= \frac{1}{2} \inf_{h \sim g \circ f^{-1}} \log K(h)$$



where $K(h) = \sup_{x \in X} K_x(h)$ quasiconf dil of h at x.

this is a metric, complete

Mod_g acts by isometries.

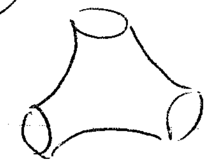
Theorem (Fricke):

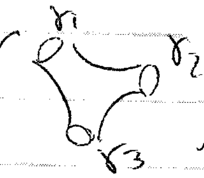
$$\mathrm{Teich}_g \simeq \mathbb{R}^{6g-6}.$$

"Proof": $\mathrm{Teich}_g \to (\mathbb{R}^+)^{3g-3} \times \mathbb{R}^{3g-3}$

$$X \mapsto \left(\ell_X(\gamma_1), \ldots, \ell_X(\gamma_{3g-3}), \theta_1(X), \ldots, \theta_{3g-3}(X)\right)$$

pants decomp.



Key: $\mathrm{Teich}\Big($  $\Big) \xrightarrow{\simeq} (\mathbb{R}^+)^3$

$$X \longmapsto \left(\ell_X(\gamma_1), \ell_X(\gamma_2), \ell_X(\gamma_3)\right)$$

<u>Theorem</u> (Fricke) Action of Mod_g on Teich_g is properly discontinuous.

<u>Cor</u>: $H^*(\mathcal{M}_g, \mathbb{Q}) \simeq H^*(\mathrm{Mod}_g, \mathbb{Q})$.

<u>Proof of the Thurston classif. - ideas</u>

Given $\varphi \in \mathrm{Mod}_g$, φ acts by isom of $(\mathrm{Teich}_g, d_{\mathrm{Teich}})$

Look at $\tau(\varphi) = \inf\limits_{x \in \mathrm{Teich}_g} d(\varphi(x), x)$

3 cases
- $\tau(\varphi) = 0$ inf realized.
- $\tau(\varphi) > 0$ and realized
- $\tau(\varphi)$ not realized.