Note Taker Checklist Form -*MSRI*

Name : Yael Algom-Kfir

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Talk Title and Workshop assigned to:
Rank 1 isometries on CAT(0) spaces and quasi-homomorphisms

Lecturer (Full name): Koji Fujiwara

Date & Time of Event: Nov 6, 2007 3:30 PM - 4:20 PM

Check List:

( ) Introduce yourself to the lecturer prior to lecture. Tell them that you will be the note taker, and that you will need to make copies of their own notes, if any.

( ) Obtain <u>all</u> presentation materials from lecturer (i.e. Power Point files, etc). This can be done either before the lecture is to begin or after the lecture; please make arrangements with the lecturer as to when you can do this.

( ) Take down <u>all</u> notes from media provided (blackboard, overhead, etc.)

( ) Gather <u>all</u> other lecture materials (i.e. Handouts, etc.)

( ) Scan all materials on PDF scanner in 2nd floor lab (assistance can be provided by Computing Staff) – Scan this sheet first, then materials. In the subject heading, enter the name of the speaker and date of their talk.

Please do **NOT** use **pencil** or colored pens other than black when taking notes as the scanner has a difficult time scanning pencil and other colors.

Please fill in the following after the lecture is done:

1. List 6-12 lecture keywords: ______

2. Please summarize the lecture in 5 or less sentences.

Once the materials on check list above are gathered, please scan ALL materials and send to the Computing Department. Return this form to Larry Patague, Head of Computing (rm 214)

$f : G \to \mathbb{R}$ is <u>quasi-homomorphism</u> if

$$\sup_{a,b \in G} \left| f(a) + f(b) - f(ab) \right| = D(f) < \infty$$

<u>homogeneous</u> if $f(a^n) = n \cdot f(a)$

$$\bar{f}(a) = \lim_{n \to \infty} \frac{f(a^n)}{n}$$ is homog.

$HQH(G) = \{$all homo. q-homo. on $G\}$

$\check{H}^1(G) = \{$all homo. on $G\}$

$\widehat{HQH}(G) = HQH(G) / H^1(G)$

$\widehat{HQH}(G) = 0$ if G amen.

<u>Th</u> (Burger-Monod)

Γ : irr lattice in s.s. Lie gp of rank ≥ 2

$\Rightarrow \widetilde{HQH}(\Gamma) = 0$

<u>Th</u> (Bestvina-F)

S : cpt ori. surface

curve graph $\curvearrowright C(S)$ δ-hyp (Masur-Minsky)

$\exists a \in) G < Mod(S)$

p. Anosov

not v. abel

$\Rightarrow \widetilde{HQH}(G)$ is ∞-dim

Cor

<u>Th</u> (Kaimanovich-Masur)

$\Gamma \not< Mod(S)$

higher rank lattice

<u>Th</u> (Farb-Masur) $f : \Gamma \to Mod(S)$

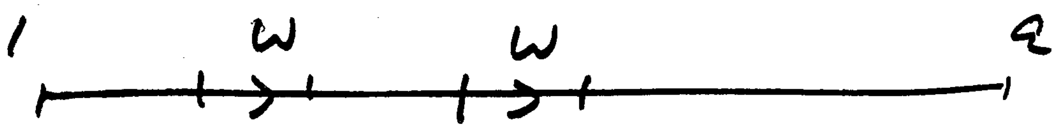
$\Rightarrow Im(f)$ finite

<u>Brooks</u> $\widetilde{HQH}(F_2)$ is ∞-dim.

Fix $w \in F$, ~~irr.~~ red. cyc red word $\neq \emptyset$.
counting func.

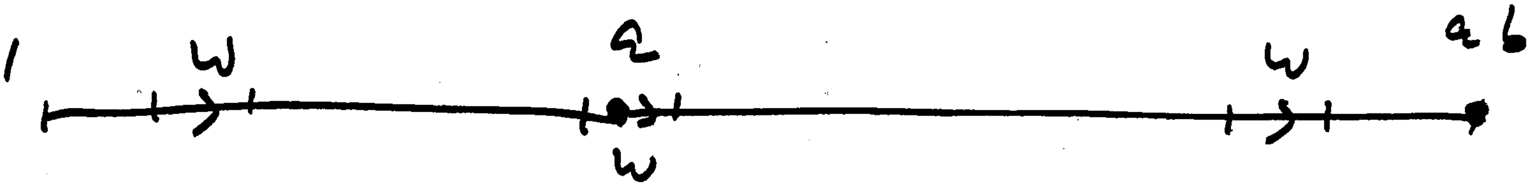
$a \in F$, $|a|$ word length

$C_w(a) = |a|_w = \max\{\#$ of w in a as subword without overlap $\}$

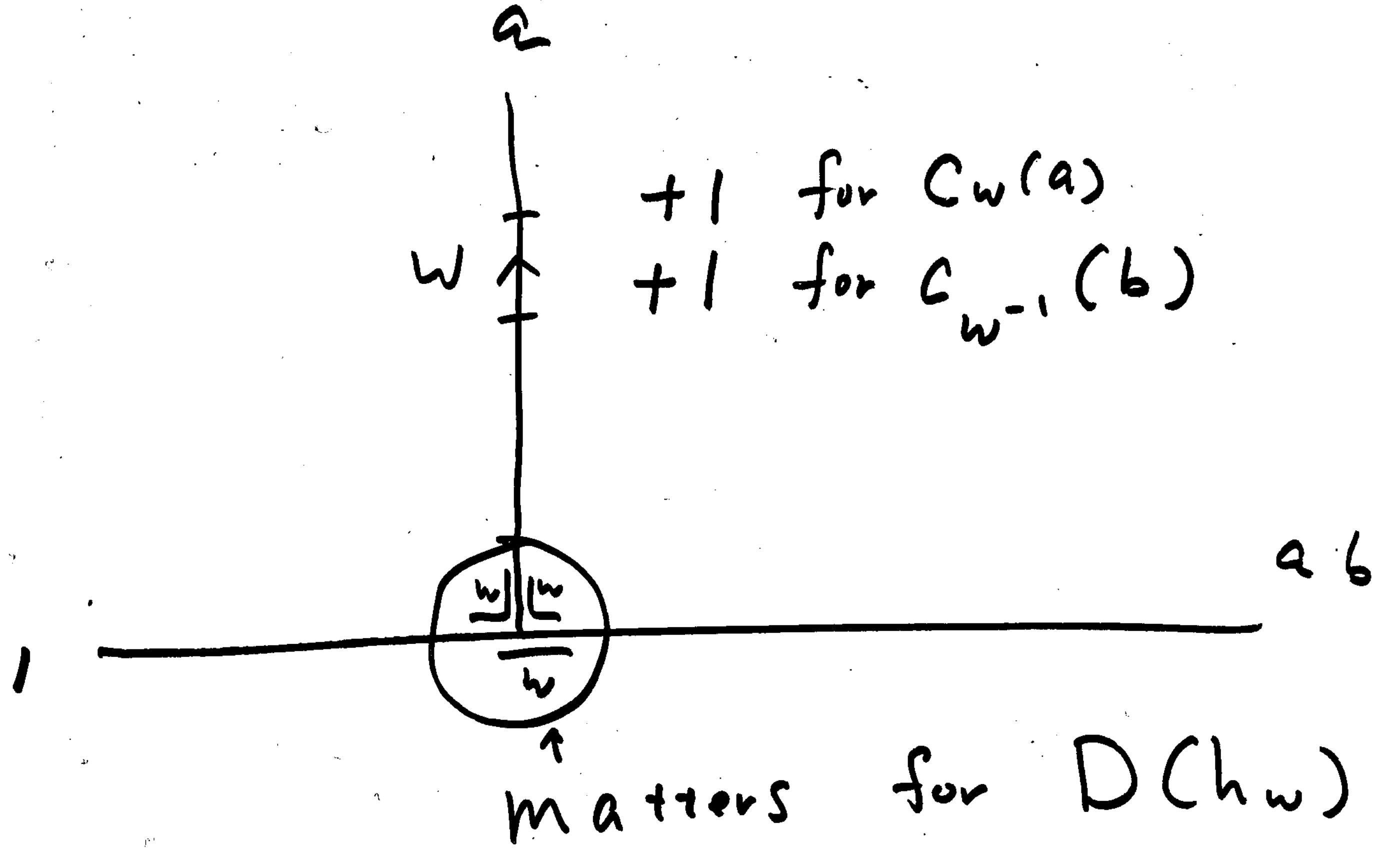


$h_w = C_w - C_{w^{-1}}$. $h_w(w^n) = n$ $(n > 0)$

<u>prop</u> $D(h_w) \leq 3$



$$0 \leq C_w(ab) - C_w(a) - C_w(b) \leq 1$$

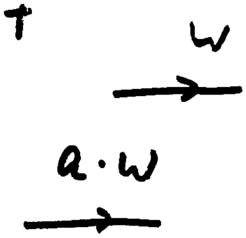

a
w
+1 for $C_w(a)$
+1 for $C_{w^{-1}}(b)$
1
ab
w
w
w
matters for $D(h_w)$

δ-hyp case

$G \curvearrowright \Gamma$ by isom, Γ δ-hyp, $x_0 \in \Gamma$

Fix $w \subset \Gamma$, good seg.

$\{ a(w) \mid a \in G \}$ G-orbit



$\alpha \subset \Gamma$ path, $|\alpha|$

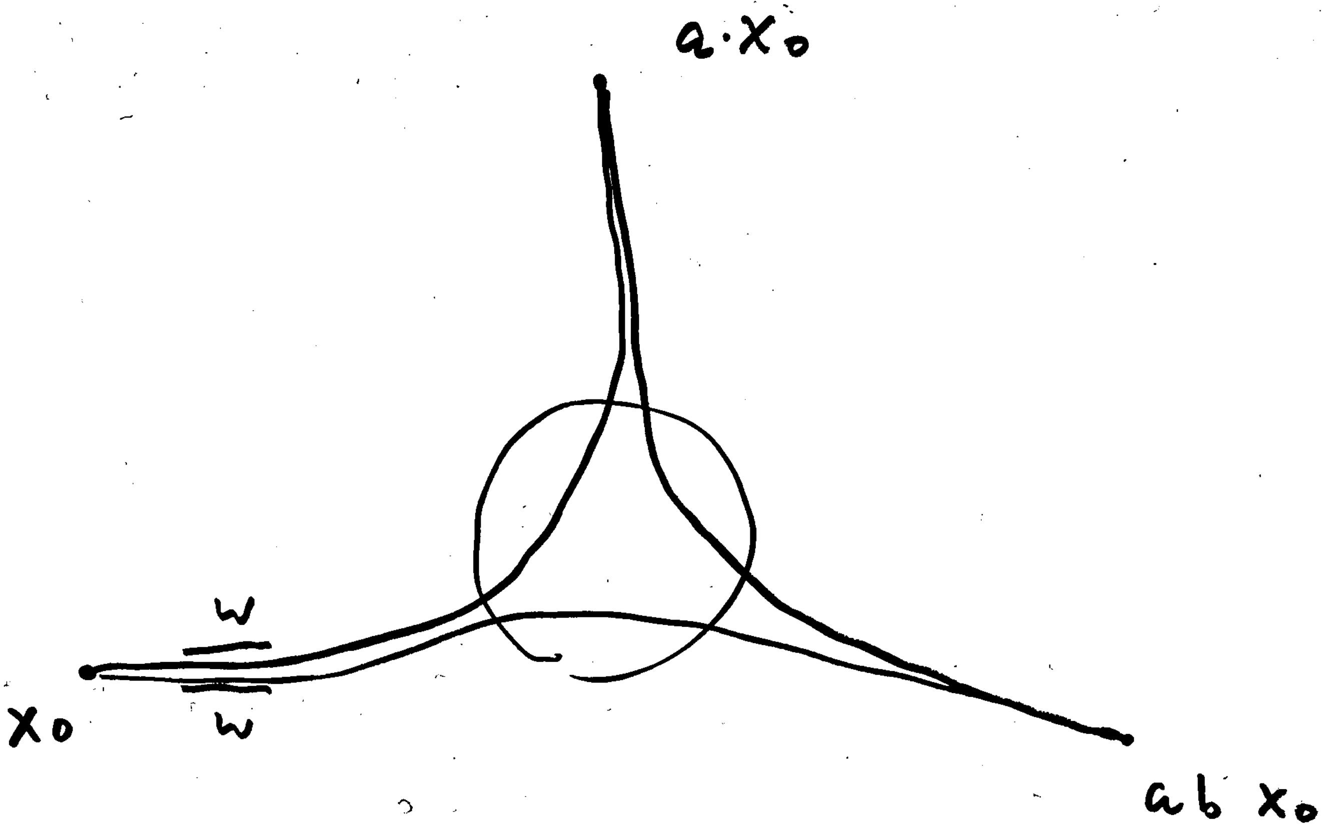
$|\alpha|_w = \max \{ \#$ of w-orbit in α without overlap $\}$

$$C_w(a) = |x_0 - a(x_0)| - \min_{\alpha} \left(|\alpha| - |\alpha|_w \right)$$

α from x_0 to $a(x_0)$

$h_w = C_w - C_{w^{-1}}$

prop Γ: δ-hyp $\Rightarrow D(h_w) < \infty$



minimizing path Δ is thin

<u>Th</u> (Epstein-F)
G: w-hyp, non-elem $\Rightarrow$ $\widetilde{HQH}(G)$ ∞-dim

$\Uparrow$

<u>Th</u> (Bestvina-F)
$G < \mathrm{Mod}(S)$ not v. abel

use $\mathrm{Mod}(S) \curvearrowright C(S)$ curve graph
δ-hyp (Masur-Minsky)

<u>Th</u> (Calegari-F)

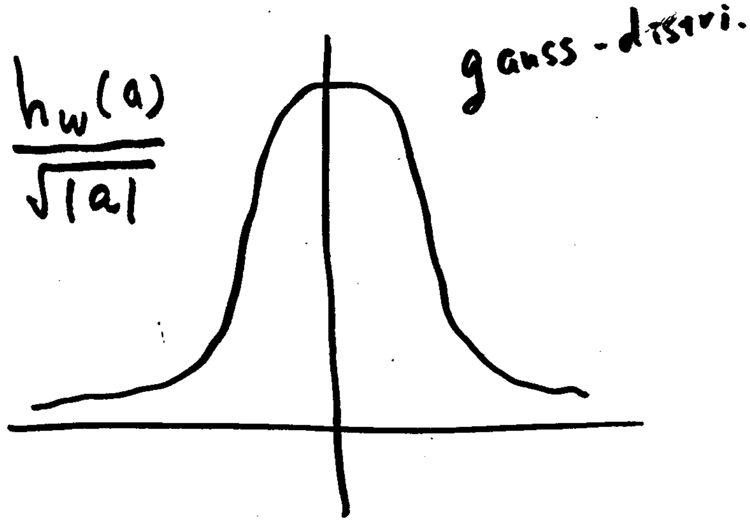
G : w-hyp, non-elem

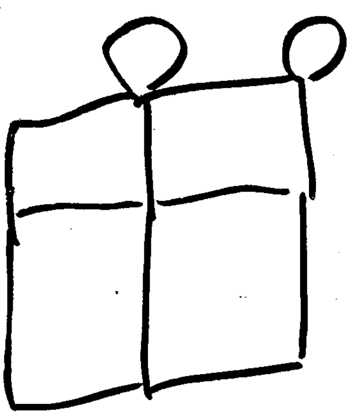
$h_w \in HQH(G)$

is a "combable function",

therefore satisfies CLT

(central limit thm).





auto str. (put value on vertex and read it off

use Markov process to get CLT

Main Th (Bestvina - F)

X : CAT(0) (or geod sp with DD + FT)

$G \curvearrowright X$ by isom, WPD
weak prop. discont.

$\exists\, a \in G$, rank-1

G not v. $\mathbb{Z}$

$\Longrightarrow \widetilde{HQH}(G)$ is ∞-dim

Cor

M : complete R-mfd, vol $< \infty$, $K \leq 0$

$G = \pi_1(M)$ irr. dim $M \geq 2$

TFAE

(1) M is loc. symm. sp of rank ≥ 2

(2) $\widetilde{HQH}(G) = 0$

pf of Cor

(1) $\Rightarrow$ (2) Burger-Monod

(2) $\Rightarrow$ (1) use Rank-rigidity Main Thm

B-contracting

$\gamma \subset X$ geod., $B > 0$

$\pi_\gamma : X \to \gamma$ projection

γ is <u>B-contracting</u> if

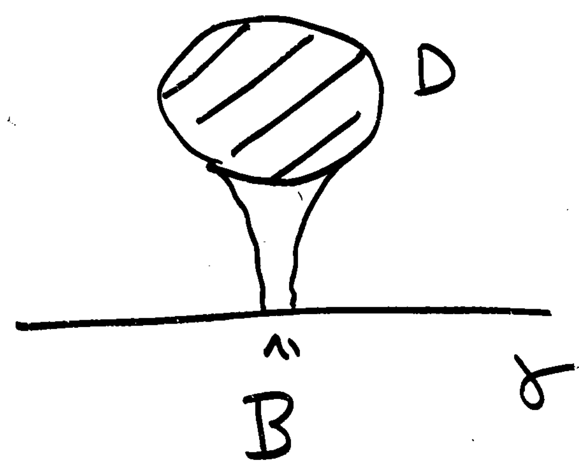
$\forall D \subset X$ metric ball, $D \cap \gamma = \emptyset$

$\operatorname{diam} \pi_\gamma(D) \leq B$

prop

X : δ-hyp

$\Rightarrow \gamma$ is 10δ-contr.



$a : X \to X$ isom is <u>rank-1</u>

(semi-simple)

if $\exists\, \gamma \subset X$ geod. "almost" a-inv.

a is transl. on γ

s.t. <u>γ is B-contr</u>

for some B.

WPD (weak proper discont.)

$x_0 \in X$

$G \curvearrowright X$ by isom.

WPD if

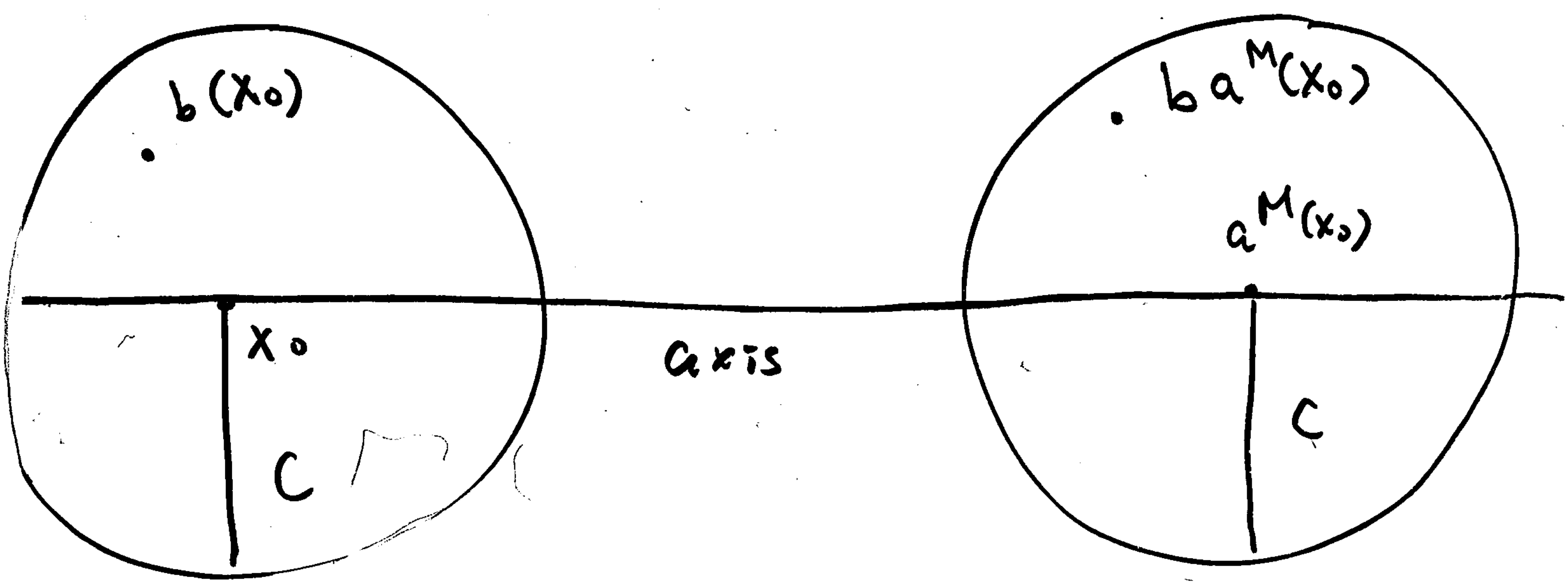
$\forall a \in G$, rank-1

$\forall C > 0$

$\exists M > 0$ s.t.

$$\{ b \in G \mid |x_0 - b(x_0)| \le C, \ |a^M(x_0) - b a^M(x_0)| \le C \}$$

is finite



Rank Rigidity (Ballmann, Eberlein, ...)

M : complete R-mfd

$K \leq 0$, vol $< \infty$

$G = \pi_1(M)$ irr.

Then either

(1) M is loc. symm. sp $\forall \gamma \subset \tilde{M}$ of rank ≥ 2 $\cap \mathbb{E}^2$

or

(2) $\exists \gamma \subset M$ cl. geod. s.t. $\tilde{\gamma} \subset \tilde{M}$ does not bound half $\mathbb{E}^2$

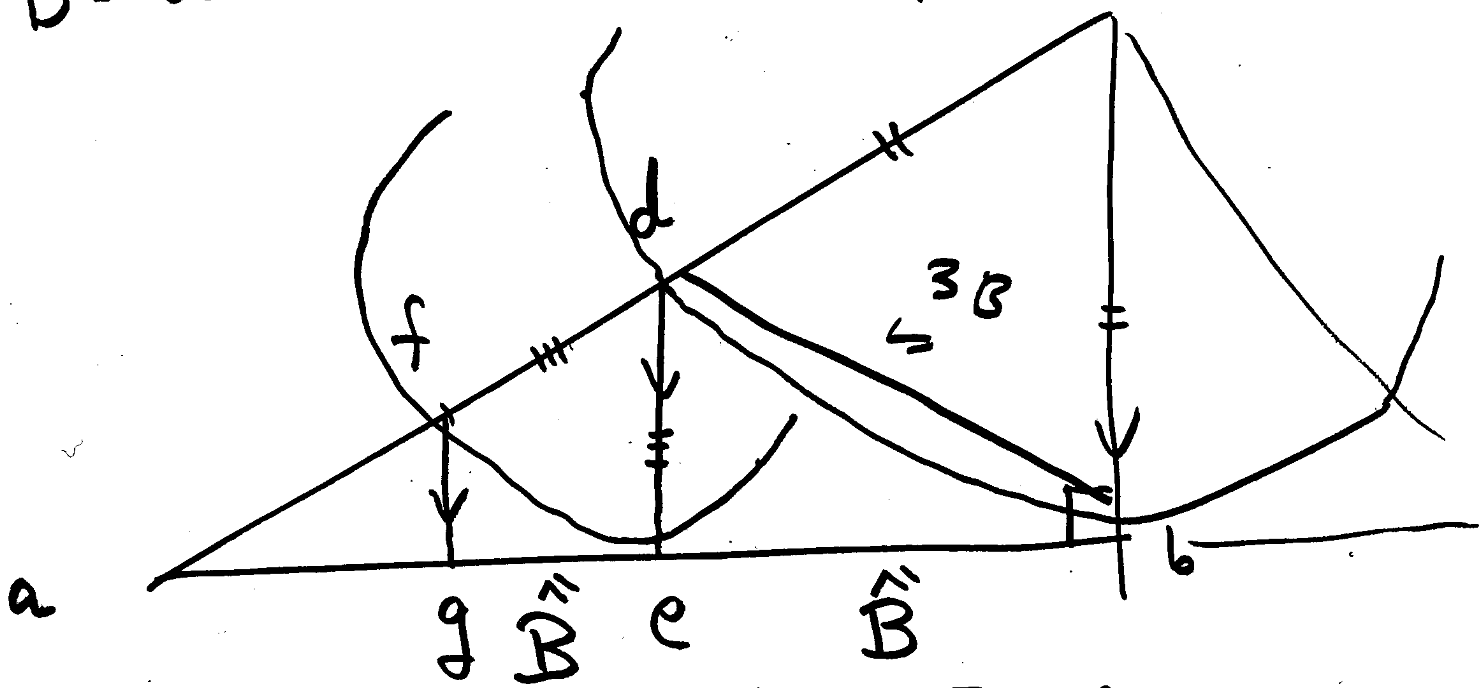
prp (2) $\Leftrightarrow$ $[\gamma] \in \pi_1(M)$ is rank-1 on $\tilde{M}$.

pf of Cor

not (1) $\xRightarrow{\text{R.R.}}$ $\exists$ rank-1 in $\pi_1(M$ $\xRightarrow{\text{Th}}$ $\widetilde{HQH} \neq 0$ not (2) //

Lemma in CAT(0),

B-contr. $\Longrightarrow$ thin Δ.



$[a,b]$ is B-contr, $b \in \pi_{ab}(c)$

Pf.) $|a-c| \leq |a-b| + |b-c|$

$\therefore |a-d| \leq |a-b|$
$\leq |a-e| + B$

$\therefore |a-d| - |a-e| \leq B$

Also, $|a-d| - |d-e| = |a-f|$
$\geq |a-g|$
$\geq |a-e| - B$

$\therefore \underbrace{|a-d| - |a-e|}_{\hat{B}} + B \geq |d-e|$

$2B \geq |d-e|$
$3B \geq |d-b|$ //

pf of Main thm

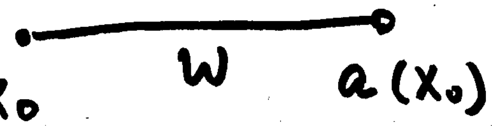
~~not(1) ⇒ not(2) ?~~
~~R.R. ⇓~~
~~∃ rank-1~~

by counting function.

$a \in G$ rank-1 isom on X

α axis $\subset X$

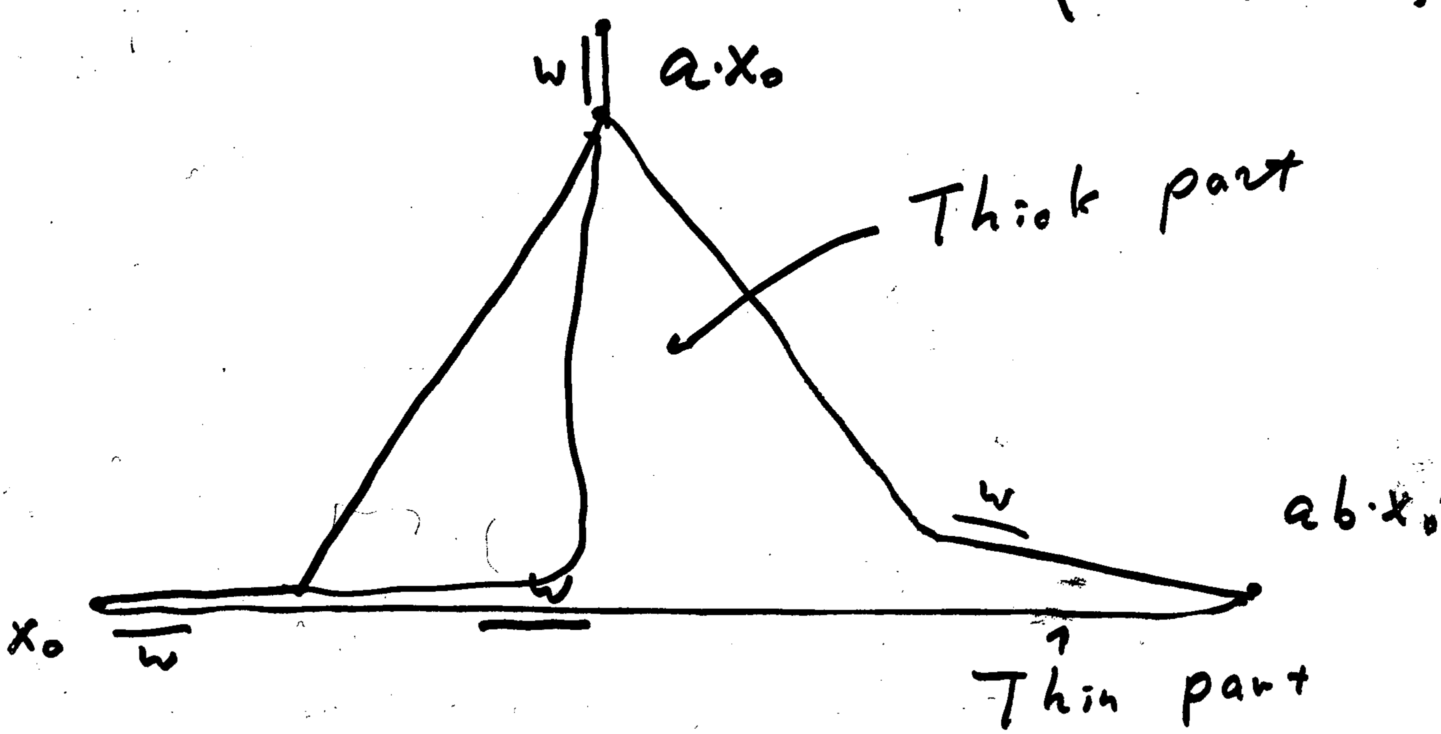
$\cup$



$c_w : G \to \mathbb{Z}$

$h_w : G \to \mathbb{Z}$

prop $D(h_w) < \infty$ $\left(\stackrel{\Rightarrow}{\widetilde{HQH}(G)} \text{ is } \infty \text{ dim} \right)$



W-orbit appears only near Thin part!

Weil-Petersson geometry

S : cpt, ori. surface

$WP(S)$: Teich. sp
w. Weil-Petersson metric
(R-mfd, $K<0$, not complete)

$Mod(S) \curvearrowright WP(S)$ by isom.

$\Downarrow$
a, p-Anosov has geod. axis

prop · $Mod(S) \curvearrowright WP(S)$ is WPD
· p-Anosov is rank. 1

Cor $G < Mod(S)$, not v. $\mathbb{Z}$

$\Downarrow$
$\exists\, a$ p-Anosov

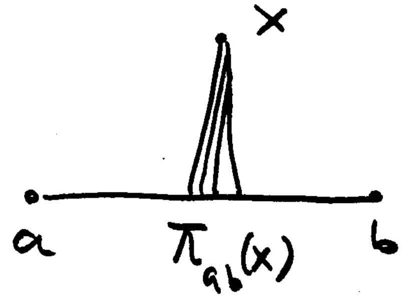
$\Rightarrow \widetilde{HQH}(G)$ is ∞-dim.

(pf) use Main Thm.

Axioms

(X, d) geod sp

$[a, b]$ geod



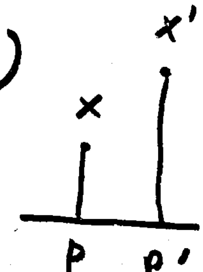
$x \in X$

$\pi_{ab}(x) \subset [a, b]$ minimizing dist. to x.

$C > 0$

(DD) $\forall p \in \pi_{ab}(x), \forall p' \in \pi_{ab}(x')$

$$|p - p'| < |x - x'| + C$$

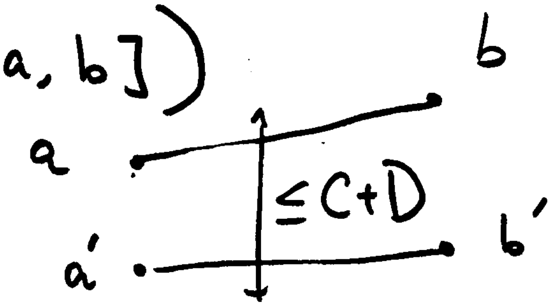


In particular, diam $\pi_{ab}(x) < C$

(FT)

$$|a - a'| \le D, \ |b - b'| \le D$$

$$\Rightarrow [a', b'] \subset N_{C+D}([a, b])$$



prop

CAT(0), δ-hyp have (DD)+(FT)