

DESCRIPTION OF PUBLICATIONS

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The articles are listed in chronological order, with most recent publications and work in progress described at the end of the document.

Published research papers

1. On tolerance sphere-of-influence graphs

(with K. Bogart) *Bull. Inst. Combin. Appl.* **24** (1998) 33–46.

DESCRIPTION: The sphere-of-influence graph $G(S)$, a type of intersection graph, on a set S of points in the plane was introduced by G.T. Toussaint. To define a sphere-of-influence graph $G(S)$ on a finite set S of vertices we first assign to each vertex in S a point in the Euclidean plane, and an open ball centered at that point with radius equal to the smallest distance between that point and any other point representing a vertex in S . Two vertices in S are adjacent if and only if their balls overlap. In the case of a sum (min) tolerance sphere-of-influence graph, we also assign each vertex a real number called a tolerance and define two vertices to be adjacent if their balls overlap by more than the sum (min) of the two tolerances. Our main theorems state that every tree is a sum tolerance sphere-of-influence graph and a min tolerance sphere-of-influence graph. The results differ slightly when considering closed balls. In contrast, not all trees are sphere-of-influence graphs.

2. On majority domination in graphs

Discrete Math., **239** (2001) 1–12.

DESCRIPTION: A majority dominating function on the vertex set of a graph $G = (V, E)$ is a function $g : V \rightarrow \{1, -1\}$ such that $g(N[v]) \geq 1$ for at least half of the vertices v in V . The weight of a majority dominating function is denoted $g(V)$ and is $\sum g(v)$ over all v in V . The majority domination number of a graph is the minimum possible weight of a majority dominating function, and is denoted $\gamma_{\text{maj}}(G)$. We determine the majority domination numbers of certain families of graphs. Moreover, we show that the decision problem corresponding to computing the majority domination number of an arbitrary disjoint union of complete graphs is NP-complete.

3. The equivariant cohomology of Hamiltonian G -spaces from residual S^1 actions

(with R. Goldin) *Math. Research Letters* **8** (2001) 67–78.

Preprint `math.SG/0107131`.

DESCRIPTION: We show that for a Hamiltonian action of a compact torus T on a compact, connected symplectic manifold M , the T -equivariant cohomology is determined by the residual S^1 action on the submanifolds of M fixed by codimension-1 subtori. This theorem allows us to compute the equivariant cohomology of certain manifolds, which have pieces that are four-dimensional or smaller. We give several examples of the computations that this allows.

4. Distinguishing chambers of the moment polytope

(with R. Goldin and L. Jeffrey) *Journal of Symplectic Geometry* **2** (2003), no. 1, 109–131.

Preprint math.SG/0302265.

DESCRIPTION: If M is a compact Hamiltonian T -space, then the combinatorics of the moment polytope Δ yields much information about the topology of M . This polytope contains facets, which correspond to points fixed by a subcircle of T , and the facets divide the moment polytope into chambers. There is an ideal associated to each point in every chamber, and our main theorem states that if the T -fixed points are isolated, then these ideals distinguish the chambers. That is, two ideals are the same if and only if the two points are in the same chamber. For flag varieties, these ideals are generated by the classes dual to permuted Schubert varieties.

5. The mod 2 equivariant cohomology of real loci

(with D. Biss and V. Guillemin) *Adv. Math.* **185** (2004) no. 2, 370–399.

Preprint math.SG/0107151.

DESCRIPTION: We extend the results of Goresky, Kottwitz and MacPherson to the real loci of symplectic manifolds. We use the combinatorics of the moment polytope to give a description of the equivariant cohomology of the real locus, using \mathbb{Z}_2 coefficients. Examples include the real points of a toric variety, and the real points of a flag variety.

6. GKM theory for torus actions with non-isolated fixed points

(with V. Guillemin) *International Math. Res. Notices* **40** (2004) 2105–2124.

Preprint math.SG/0308008.

DESCRIPTION: In this paper, we consider compact symplectic manifolds equipped with Hamiltonian torus actions. We make an additional assumption on the isotropy weights at the torus-fixed points. This allows us to give a combinatorial description of the one-skeleton of the manifold, and hence a combinatorial description of the torus-equivariant cohomology of the manifold.

7. Real loci of symplectic reductions

(with R. Goldin) *Trans. AMS*, **356** (2004), no. 11, 4623–4642.

Preprint math.SG/0209111.

DESCRIPTION: The **real locus** of a symplectic manifold is the fixed set of an anti-symplectic involution. It is a Lagrangian submanifold. Examples of real loci include the set of real points on a complex projective variety. We extend the notion of quotient to real loci, and we relate the topology of the quotient to the topology of the original space, extending standard results of symplectic geometry to real loci.

8. Conjugation spaces

(with J-Cl. Hausmann and V. Puppe) *Algebr. Geom. Topol.* **5** (2005) 923-964.

Preprint [math.AT/0409305](#).

DESCRIPTION: We consider spaces equipped with an involution, and study the ordinary and equivariant cohomology of those spaces and of the fixed point set of the involution. These techniques allow us to generalize many results, including results from the study of real loci in symplectic geometry.

9. Computation of generalized equivariant cohomologies of Kac-Moody flag varieties

(with M. Harada and A. Henriques) *Adv. Math.*, **197** (2005) No. 1, 198–221

Preprint [math.AT/0409305](#).

DESCRIPTION: Let G be a topological group, and X a stratified G space equipped with a stratification-compatible G -action. Let E_G^* be a generalized equivariant cohomology theory. For these spaces, we give a combinatorial description of $E_G^*(X)$ as a subring of $\prod E_G^*(F_i)$, where the F_i are certain invariant subspaces of X . Our main examples are the flag varieties \mathcal{G}/\mathcal{P} of Kac-Moody groups \mathcal{G} , with the action of the torus of \mathcal{G} . In this context, the F_i are the T -fixed points and E_G^* is a T -equivariant complex oriented cohomology theory, such as H_T^* , K_T^* or MU_T^* . We compute several explicit examples.

This is a substantially rewritten version of the preprint [math.DG/0402079](#).

DESCRIPTION OF [math.DG/0402079](#): In this paper, we study torus actions on cell complexes. We extend results from symplectic geometry to this setting to compute the T -equivariant cohomology of the cell complex. Our main examples include flag varieties \mathcal{G}/\mathcal{P} of Kac-Moody groups \mathcal{G} , with the action of the torus of \mathcal{G} . This paper is subsumed in the published version, but the two papers are nearly disjoint. The techniques are different, and as such, we posted the published version as a separate preprint.

10. The equivariant cohomology of hypertoric varieties and their real loci

(with M. Harada) *Communications in Analysis and Geometry*, **13** (2005) No. 3 645–677.

Preprint [math.SG/0405422](#).

DESCRIPTION: In this paper, we study hypertoric varieties, which are the hyper-Kähler analogue of toric varieties. These are hyper-Kähler quotients of affine quaternionic space. We give a description of the equivariant cohomology ring of a hypertoric variety as a subring of a direct sum of polynomial rings. We relate our results to the description given by Konno in terms of generators and relations. Finally, we discuss the real loci of hypertoric varieties and compute their cohomology (with \mathbb{Z}_2 coefficients).

11. A GKM description of the equivariant cohomology ring of a homogeneous space

(with V. Guillemin and C. Zara) *J. Algebraic Combin.* **23** (2006) no. 1, 21–41.

Preprint math.SG/0112184.

DESCRIPTION: Let T be a torus of dimension $n > 1$ and M a compact T -manifold. M is a GKM manifold if the set of zero dimensional orbits in the orbit space M/T is zero dimensional and the set of one dimensional orbits in M/T is one dimensional. In this paper, we prove that every compact homogeneous space M of non-zero Euler characteristic is of GKM type and show that the graph associated with M encodes geometric information about M as well as topological information. For example, from this graph one can detect whether M admits an invariant complex structure or an invariant almost complex structure.

12. Connectivity properties of moment maps on based loop groups

(with M. Harada, L. Jeffrey and A-L. Mare) *Geom. Topol.* **10** (2006), 1607–1634.

Preprint math.SG/0503684.

DESCRIPTION: For a compact, connected, simply-connected Lie group G , the loop group LG is the infinite-dimensional Hilbert Lie group consisting of maps from S^1 to G . The geometry of LG and its homogeneous spaces is important in representation theory and has been extensively studied. The space of based loops $\Omega(G)$ is an example of a homogeneous space of LG and has a natural Hamiltonian $T \times S^1$ action, where T is the maximal torus of G . We study the moment map μ for this action, and in particular prove that its regular level sets are connected. This result is as an infinite-dimensional analogue of a theorem of Atiyah that states that the preimage of a moment map for a Hamiltonian torus action on a compact symplectic manifold is connected. In the finite-dimensional case, this connectivity result is used to prove that the image of the moment map for a compact Hamiltonian T -space is convex. Thus our theorem can also be viewed as a companion result to a theorem of Atiyah and Pressley, which states that the image $\mu(\Omega(G))$ is convex. We also show that for the energy functional E , which is the moment map for the S^1 rotation action, each non-empty preimage is connected.

13. Orbifold cohomology of torus quotients

(with R. Goldin and A. Knutson) *Duke Math. J.* **139** (2007) no. 1, 89–139.

Preprint math.SG/0502429.

DESCRIPTION: We show how to compute the orbifold cohomology of orbifolds that arise as abelian symplectic quotients. We define the **inertial cohomology** of a Hamiltonian T -space, and prove that this surjects, as a ring, onto the orbifold cohomology of the quotient. The technical heart of the result is the Kirwan surjectivity theorem in symplectic geometry. The inertial cohomology ring is easy to compute. We provide several explicit examples, and relate our work to the recent work of many others.

14. Orbifold cohomology of abelian symplectic reductions and the case of weighted projective spaces.

Contemp. Math.: Poisson geometry in mathematics and physics, **450** (2008) 127–146.

Preprint [arXiv:0704.0257](https://arxiv.org/abs/0704.0257).

DESCRIPTION: The aim of this paper is two-fold: first to advertise the ease of computation of cohomological invariants in the symplectic category; and second to give an account of some new computations for weighted projective spaces. We start with a brief exposition of how orbifolds arise in the symplectic category, and discuss the techniques used to understand their topology. We then show how these results can be used to compute the Chen-Ruan orbifold cohomology ring of abelian symplectic reductions. We conclude by comparing the several rings associated to a weighted projective space. We make these computations directly, avoiding any mention of a stacky fan or of a labeled moment polytope.

15. Torsion and abelianization in equivariant cohomology

(with R. Sjamaar) *Transf. Groups* **13** (2008) no. 3/4, 585–616.

Preprint [math.AT/0607069](https://arxiv.org/abs/math/0607069).

DESCRIPTION: Let X be a topological space upon which a compact connected Lie group G acts. It is well-known that the equivariant cohomology $H_G^*(X; \mathbb{Q})$ is isomorphic to the subalgebra of Weyl group invariants of the equivariant cohomology $H_T^*(X; \mathbb{Q})$, where T is a maximal torus of G . This relationship breaks down for coefficient rings \mathbf{k} other than \mathbb{Q} . Instead, we prove that under a mild condition on \mathbf{k} the algebra $H_G^*(X; \mathbf{k})$ is isomorphic to the subalgebra of $H_T^*(X; \mathbf{k})$ annihilated by the divided difference operators.

Manuscripts accepted for publication

16. Act globally, compute locally: group actions, fixed points, and localization

Contemp. Math., to appear.

Preprint [arXiv:0710.5295](https://arxiv.org/abs/0710.5295).

DESCRIPTION: Localization is a topological technique that allows us to make global equivariant computations in terms of local data at the fixed points. For example, we may compute a global integral by summing integrals at each of the fixed points. Or, if we know that the global integral is zero, we conclude that the sum of the local integrals is zero. This often turns topological questions into combinatorial ones and vice versa. This article features several instances of localization that occur at the crossroads of symplectic and algebraic geometry on the one hand, and combinatorics and representation theory on the other. The examples come largely from the symplectic category, with particular attention to toric varieties. In the spirit of the 2006 International Conference on Toric Topology at Osaka City University, the main goal of this exposition is to exhibit toric techniques that arise in symplectic geometry.

Preprints**17. Conjugation spaces and edges of compatible torus actions.**

(with J-Cl. Hausmann)

Preprint `arXiv:0807.3289`.

DESCRIPTION: Duistermaat introduced the concept of the **real locus** of a Hamiltonian manifold. In that and in others' subsequent works, it has been shown that many of the techniques developed in the symplectic category can be used to study real loci, so long as the coefficient ring is restricted to the integers modulo 2. It turns out that these results seem not necessarily to depend on the ambient symplectic structure, but rather to be topological in nature. This observation prompts the definition of **conjugation space** in a paper of the two authors with V. Puppe. Our main theorem in this paper gives a simple criterion for recognizing when a topological space is a conjugation space.

18. How is a graph like a manifold?

(with E. Bolker and V. Guillemin) Currently being turned into a much more substantial monograph, with (in addition) C. Zara.

Preprint `math.CO/0206103`.

DESCRIPTION: In this article, we discuss some classical problems in combinatorics which can be solved by exploiting analogues between graph theory and the theory of manifolds. One well-known example is the McMullen conjecture, which was settled twenty years ago by Richard Stanley by interpreting certain combinatorial invariants of convex polytopes as the Betti numbers of a complex projective variety. Another example is the classical parallel redrawing problem, which turns out to be closely related to the problem of computing the second Betti number of a complex compact $(\mathbb{C}^*)^n$ -manifold.

We have now added substantially to this preprint with the goal of turning it into a much more substantial (125–150 pages) monograph describing the current state-of-the-art in GKM theory.

Conference proceedings, expository articles and theses**19. Kirwan surjectivity for preorbifold cohomology**(with R. Goldin and A. Knutson) *Cohomological Aspects of Hamiltonian Group Actions*, Mathematisches Forschungsinstitut Oberwolfach Report no. 20 (2004) 36–39.

DESCRIPTION: We announce results that allow us effectively to compute the orbifold cohomology of orbifolds that arise as abelian symplectic quotients. The technical heart of the proofs is the Kirwan surjectivity theorem in symplectic geometry. These results are proved in [13].

20. A combinatorial card trick

(with S. Simonson) PRIMUS, **XIII** no. 3 (2003), 248–269.

DESCRIPTION: We present a card trick that can be used to review or teach a variety of topics in discrete mathematics. We address many subjects, including permutations, combinations, functions, graphs, depth first search, the pigeonhole principle, greedy algorithms, and concepts from number theory. Moreover, the trick motivates the use of computers in mathematical research. The ultimate solution to the card trick makes use of Hall's Distinct Representative Theorem. The events described in this paper occurred when I was teaching at ArsDigita University.

21. Equivariant Cohomology, Homogeneous Spaces and Graphs

Ph.D. Thesis, MIT, 2002.

DESCRIPTION: The focus of this thesis is manifolds with group actions, in particular symplectic manifolds with Hamiltonian torus actions. We investigate the relationship between the equivariant cohomology of the manifold M and the fixed point data of the torus action. We are interested in understanding the topology of the space of T -orbits in M . In particular, we explore aspects of this topology which are determined by data from the image of a moment map $\Phi : M \rightarrow \mathfrak{t}^*$ associated to the Hamiltonian action. First, we exploit current theory to understand the geometry of homogeneous spaces with non-zero Euler characteristic. Second, we discuss several results to enlarging the class of manifolds to which this theory applies.

In the projects described in this thesis, we apply a theorem of Goresky, Kottwitz, and MacPherson to the case of homogeneous spaces by studying the combinatorics of their associated graphs. Next, we describe how to weaken the hypotheses of the GKM theorem. The spaces to which the GKM theorem applies must satisfy certain dimension conditions; however, there are many manifolds M with naturally arising T -actions that do not satisfy these conditions. We allow a more general situation, which includes some of these cases. Finally, we find a theory identical to the GKM theory in a setting suggested by work of Duistermaat. As in the GKM situation, this theory applies only when the space satisfies certain dimension conditions.

22. Tolerance Sphere-of-Influence Graphs

Senior Honors Thesis, Dartmouth College, 1997.

DESCRIPTION: In the 1980's, G.T. Toussaint introduced the sphere of influence graph, a type of intersection graph, in the setting of pattern recognition and computer vision. His interest stemmed from a computer science point of view, and so the original questions asked related to algorithms used to create sphere of influence graphs. In this thesis, we first present an overview of sphere of influence graphs. Then we introduce the concept of tolerance to these graphs. We make use of two different tolerance functions, sum and min tolerance. Many of our results for sum tolerance are immediate, however we present constructive proofs which will give us insight into the min tolerance model. We give some history of older problems relating to our sum tolerance model. Then we define our new tolerance model, and attempt to prove some of the same results.