Exercise 1. Durrett Exercise 8.2.4. (i) Suppose $f(t) > 0$ for all $t > 0$. Use Theorem 8.2.3 to conclude that $\limsup_{t \downarrow 0} B(t)/f(t) = c$, $\mathbb{P}^0$ almost surely where $c \in [0, \infty]$ is a constant. (ii) Show that if $f(t) = \sqrt{t}$ then $c = \infty$, so with probability one Brownian paths are not Hölder continuous of order $1/2$ at 0.

If you chose to have this problem graded last homework then you need not turn it in again.

Exercise 2. Durrett 8.3.1. Let $S$ be a stopping time and let $S_n = ([2^n S] + 1)/2^n$ where $[x]$ is the least integer less than or equal to $x$. Show that $S_n$ is a stopping time.

Exercise 3. Durrett 8.3.7. Let $S$ be a stopping time. Show that $B_S$, Brownian motion is $\mathcal{F}_S$ measurable. Hint: using $S_n$ as in the previous exercise may prove useful.

Exercise 4. Durrett 8.3.2. If $S, T$ are stopping times show that $S \land T$, $S \lor T$, and $S + T$ are also stopping times. The case where $T = t$ will be a reoccurring stopping time the rest of the semester.

Exercise 5. Durrett 8.4.1. Generalize the proof of the reflection principle to show that

$$\mathbb{P}_0(T_a < t, \ u < B_t < v) = \mathbb{P}_0(2a - v < B_t < 2a - u).$$

Where $u < v \leq a$ and $T_a = \inf\{t : B_t = a\}$.

Exercise 6. Durrett 8.4.2. Recall $R = \inf\{t > 1 : B_t = 0\}$ the time of the first zero after time one. Show that $R$ has probability density

$$\mathbb{P}_0(R = 1 + t) = \frac{1}{\pi t^{1/2}(1 + t)}.$$