
2. Find the Fourier transform of \( f(x) = 1/(1 + x^2) \). (You may of course use the theorem on Fourier transforms proved in class or any other method.)

3. (i) Show that \( \tan z = \sin z / \cos z \) is holomorphic in

\[ \Omega = \mathbb{C} \setminus \{(k + 1/2)\pi | \ k \in \mathbb{Z}\} \]

and maps the vertical strip \( \Omega_1 = \{ z \in \mathbb{C} | |\text{Re} \ z| < \frac{1}{2} \pi \} \) biholomorphically onto \( \Omega_2 = \mathbb{C} \setminus \{ t | t \in \mathbb{R}, |t| \geq 1 \} \).

(ii) Show that the function \( w \mapsto \frac{1}{2\pi i} \text{Log} \frac{i - w}{i + w} \) maps \( \Omega_2 \) into the region where \( \text{Log} \) is holomorphic and that the inverse of \( \tan \Omega_1 \) is given by

\[ w \mapsto \frac{1}{2\pi i} \text{Log} \frac{i - w}{i + w}. \]

4. (i) Find a biholomorphic map from the strip \( \{ z \in \mathbb{C} | 0 < \text{Re} \ z < 1 \} \) onto the open unit disc \( D = D(0, 1) \).

(ii) Find a biholomorphic map from the crescent \( D(2i, 2) \setminus \hat{D}(i, 1) \) onto \( D \).

(First try to map the crescent onto a strip by a Möbius transformation.)

5. (i) Show that \( f(z) = (z + 1/z)/2 \) maps \( \Omega_1 = D(0, 1) \setminus \{ z \in \mathbb{R} | -1 < z \leq 0 \} \) biholomorphically onto \( \Omega_2 = \mathbb{C} \setminus \{ z \in \mathbb{R} | z \leq 1 \} \).

(ii) Deduce that \( \cos \) maps the halfstrip \( \{ z \in \mathbb{C} | |\text{Re} \ z| < \pi, \text{Im} \ z > 0 \} \) biholomorphically onto \( \Omega_2 \).