1. Let \( f(x) = (\pi - |x|)^2 \) for \(-\pi \leq x \leq \pi\). Prove that
\[
f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos nx.
\]
In what norm does the series converge? Now use Parseval to evaluate \( \sum_{n \geq 1} \frac{1}{n^4} \).

2. A subset \( A \) of a real vector space \( E \) is convex if for every \( x \) and \( y \) in \( A \) the line segment joining \( x \) to \( y \) is contained in \( A \), i.e. \( tx + (1-t)y \in A \) for \( 0 \leq t \leq 1 \). Let \( \| \cdot \| \) be a norm on \( E \).
   (a) Suppose that \( A \) is convex. Prove that \( \bar{A} \), the closure of \( A \), is convex.
   (b) Let \( A = B_\varepsilon(x) \) be an open ball in \( E \). Show that \( A \) is convex.

3. Recall that for \( p \geq 1 \) the \( L^p \)-norm of \( f \in C([0, 1], R) \) is defined by
\[
\|f\|_p = \left( \int_0^1 |f(x)|^p \, dx \right)^{1/p}.
\]
For \( n \geq 1 \) define \( f_n \) by \( f_n(x) = 1 - nx \) if \( 0 \leq x \leq 1/n \) and \( f_n(x) = 0 \) if \( 1/n \leq x \leq 1 \). Show that \( \{f_n\}_{n \geq 1} \) converges to the zero function in the \( L^p \)-norm. Does \( f_n(x) \to 0 \) pointwise?