4130 HOMEWORK 7

Due Tuesday April 13

(1) Let $D \subset \mathbb{R}$. Let $f, g : D \to \mathbb{R}$ and let $a$ be a cluster point of $D$. Suppose
\[ \lim_{x \to a} f(x) = L \text{ and } \lim_{x \to a} g(x) = M. \]
Show that $\lim_{x \to a} f(x)g(x) = LM$.

(2) Section 5.2.4 Exercise 3.

(3) Section 5.2.4 Exercise 4.

(4) Let $U$ be an open subset of $\mathbb{R}$ and suppose $f : U \to U$ is a $C^1$ bijection and that
$f'(x) \neq 0$ for all $x \in U$.
(a) Use the Inverse Function Theorem to show that the inverse function $f^{-1} : U \to U$
is also $C^1$.
(b) Give an example to show that $f^{-1}$ may not be $C^1$ if $f'(x) = 0$ for some $x \in U$.

(5) Section 5.4.6 Exercise 22(b). (Hint: let $y = p_1 x_1 + \cdots + p_n x_n$. For each $1 \leq i \leq n$,
estimate $f(x_i)$ using the Taylor expansion of $f$ about the point $y$, applying the
Lagrange Remainder Theorem from page 188.)