MATH 2310 QUIZ

Friday 2 October 2009. You have 50 minutes. No calculators are permitted. Please show all working.

(1) True or False? (Explain your answer.)
   (a) No linear system has exactly two solutions.
   (b) If $A$ is any $n \times n$ matrix, then the matrix $I_n - AA^T$ is symmetric. (Here, $I_n$ denotes the identity matrix of size $n \times n$.)

(2) Let

\[
A = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad
B = \begin{bmatrix} 2 & 4 & 6 \\ 0 & 1 & 0 \end{bmatrix}, \quad
C = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 0 & 2 \end{bmatrix}
\]

Calculate each of the following. If it is not defined, say so.
   (a) $A(C + B^T)$
   (b) $B^{-1}$

(3) Let $X = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$.
   (a) Show that $X^2 - 7X + I_2 = 0$.
   (b) Determine whether $X$ is invertible, and find $X^{-1}$ if it exists.

(4) Let

\[
A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 9 \\ 3 & 1 & 12 \end{bmatrix}, \quad
b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
\]

   (a) Find all solutions to the linear system $Ax = b$.
   (b) Is $A$ invertible? Explain why or why not.

(5) Determine whether the matrix

\[
A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -2 \\ 0 & -1 & 2 \end{bmatrix}
\]

is invertible, and find $A^{-1}$ if it exists.

[END]