Math 4530 — Topology. Homework 10

Due in class on 10th November, 2009.
Please declare any collaborations with classmates; if you find solutions in books or on–line, acknowledge your sources — in either case, write your answers in your own words.
Please attempt all questions and justify your answers.

1. [Munkres] Suppose \( p : E \to B \) is a covering map and \( B \) is connected. Prove that if \( p^{-1}(\{b\}) \) has \( n \) points for some \( b \in B \), then \( p^{-1}(\{b\}) \) has \( n \) points for every \( b \in B \). [Such a \( p : E \to B \) is called an \( n \)-sheeted covering of \( B \).]

2. [For class discussion]
   (a) Let \( \Sigma_g \) denote the closed orientable surface of genus \( g \) (that is, the sphere with \( g \) handles). Show that for all \( n \geq 1 \) and \( m \geq 0 \), there is an \( n \)-sheeted covering \( \Sigma_{mn+1} \to \Sigma_{m+1} \).
   *Hints.* Try the cases \( m = 0 \) and \( m = 1 \) first. The key is picturing the surfaces the right way.
   (b) Show that there is a 2–sheeted covering \( T^2 \to K \) of the Klein bottle by the torus.

3. [For class discussion]
   (a) Show that covering maps are open maps — that is, if \( p : E \to B \) is a covering map and \( U \subseteq E \) is open, then \( p(U) \) is open.
   (b) Show that covering maps are quotient maps. Is the converse true?
   (c) Show that covering maps are homeomorphisms if and only if they are bijective.

4. [For class discussion]
   (a) Draw a connected 3–sheeted covering of figure–8 space.
   (b) Draw a simply connected covering of figure–8 space.

5. Prove that for \( n > 2 \), \( \mathbb{R}^2 \) and \( \mathbb{R}^n \) are not homeomorphic.
   *Hint.* Consider removing a point from each and make sure you fully justify your answer.

TRR