Math 192, Prelim 2, April 10, 2008

You are allowed one sheet of paper. You are NOT allowed calculators or the text. SHOW ALL WORK!

1) a) (5 points) Find all local maxima, local minima and saddle points of the function $-3x^2+8xy+3y^2$.
b) (7 points) Find all extrema of the function $x+2y+3z$ subject to the constraint $3x^2+2y^2+z^2=102$.

2) a) (7 points) Sketch the region of integration for the function $\int_{-2}^{0}\int_{-\sqrt{4-x^2}}^{0}\frac{1}{1+x^2+y^2}\,dy\,dx$ and compute the integral.
b) (6 points) Integrate the function $x^2\sqrt{y^4+1}$ over the triangle in the xy-plane with vertices $(0,0)$, $(2,2)$ and $(0,2)$.

3) The questions below are true/false. Either write the entire word TRUE, the entire word FALSE or leave it blank. No work is required for this problem. A correct answer is worth 4 points. A blank answer is worth 0 points. A wrong answer is worth -5 points, so **don't guess!**
a) $(+4/-5$ points) A function $f(x,y)$ with continuous second partial deriviatives and one critical point *must* have a saddle point.
b) $(+4/-5$ points) If we insist that $0\le\theta<2\pi$ and $0<r<\infty$, then there are *exactly* two ways to write any point in the plane (other than the origin) in polar coordinates.
c) $(+4/-5$ points) The vector field $\vec{\nabla}(x^2y+e^{z+xy}-\sin^2(x+2y+z))$ is conservative.
d) $(+4/-5$ points) $\int_0^1\int_0^{\pi/2}\int_0^{\sqrt{1+z^2}} f(r,\theta,z)r\,dr\,d\theta\,dz$ is the integral of the function f in cylindrical coordinates over that part of the solid sphere centered at the origin and of radius 1 that lies in the first octant.

4) Let D be the region bounded above by the paraboloid $z=2-x^2-y^2$ and below by the cone $z=\sqrt{x^2+y^2}$.
a) (8 points) Set up triple integrals in Cartesian and cylindrical coordinates to determine the volume of this region.
b) (5 points) Evaluate one of these integrals to find the volume.

5) a) (5 points) Find the equation for the line tangent to the curve $x^4+y^4=1$ at the point (a,b).
b) (5 points) Now assume (a,b) lies in the first quadrant. Determine the area of the region in the first quadrant lying below the line found in (a).

6) a) (6 points) Integrate the function x over that part of the solid sphere of radius R where $z\le 0$ and $x,y\ge 0$.
b) (6 points) Let $C>0$ be a fixed constant. Find the volume of the region in the first octant bound by the planes $y=C$, $x+z=C$ and $x+y=C$.

7) a) (6 points) Let C be the line segment from $(0,0,0)$ to $(1,2,3)$. Find $\int_C (xyz^2)ds$.
b) (6 points) Find the flux and flow of $\vec{F}=y^2\vec{i}-x^2\vec{j}$ over the counterclockwise unit circle centered at the origin.

8) a) (6 points) Let $\vec{F}=x\vec{i}+y\vec{j}$. Find $\int_C \vec{F}\cdot d\vec{r}$ where C is the path that consists of the four directed line segments in the xy-plane from $(1,0)$ to $(1,1)$, $(1,1)$ to $(0,2)$, $(0,2)$ to $(-1,1)$ and $(-1,1)$ to $(-1,0)$.
b) (6 points) Let $\vec{F}(x,y,z)=2xy^3\vec{i}+3y^2x^2\vec{j}+P(x,y,z)\vec{k}$. Explain, with your reasoning, for which scalar functions $P(x,y,z)$ the vector field $\vec{F}(x,y,z)$ is conservative.