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Vibratory Fruit Harvesting: A Non-linear Theory of Fruit-stem Dynamics*

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The non-linear, normal mode free vibrations of the fruit and stem are analysed as a double pendulum with elastic couplings. The in-phase mode is shown to agree closely with the linear analysis; the out-of-phase mode is shown to be strongly amplitude-dependent. Numerical integration of the coupled non-linear equations of motion indicates that the approximate analytical results are valid for angles as large as one rad. Calculations for apples, cherries, grapefuit, peaches, lemons, oranges, olives, coffee, plums and raspberries are presented and some implications for shaker equipment operation are indicated; e.g. it is conjectured that for fruits to be harvested without stems attached, a pulsating frequency will produce more efficient fruit detachment of the desired type than would continuous shaking.

1. Introduction

The vibratory behaviour of fruit during the mechanical harvesting operation has been described by Cooke and Rand[1] in terms of the linearized equations of motion for a planar double pendulum with torsional elastic couplings. The next logical extension of this analysis is a consideration of the effects of the non-linearity inherently present in the equations of motion. This sequel to the linear analysis will, therefore, examine the non-linear motion of the free vibration of the fruit-stem system.

The linear theory included a discussion of the effect of the direction and frequency of shaking upon fruit detachment during vibratory harvesting. The fruit and stem were excited by periodic simultaneous horizontal and vertical motion of the supporting structure. The desired conditions for fruit detachment were assumed to be that of relative bending motion between the stem and supporting structure (e.g. apples with stems) or between the fruit and the stem (e.g. grapefruits without stems). Depending upon the anticipated processing, cherries may be harvested with or without stems attached. This criterion of cyclical, relative bending contrasts with the frequently-used assumption of a critical tensile force (or impulse) which must be produced. The latter assumption is expressed sometimes simply as a desire to produce vigorous tree vibrations. On the other hand, the greatest relative bending of the stem should result in a region of dynamic instability for the fruit and stem. Accordingly, the natural frequency which corresponds to the desired mode shape should be utilized. Whenever a vertical component of excitation of the supporting structure exists, the forcing frequency should be twice the selected natural frequency (rather than one of the other narrower unstable regions for the non-homogeneous Mathieu equation). The appendix of the present paper supplements Reference (1) by proving this result.

Fig. 1 shows 2 of the principal modes of vibration as described by Diener *et al.*[2] The pendulum mode depicts greater bending θ at the upper stem end than at the lower $(\phi-\theta)$, and the fruit and stem are usually in phase. The tilting mode exists when the relative bending is greater at the lower stem end than at the upper, and the fruit and stem are usually out of phase. The twisting mode involves rotation ψ of the fruit with respect to the axis passing from the stem to calyx end of the fruit.

Papers on mechanical fruit harvesting by Garman *et al.*[3] and Liang *et al.*[4] have been presented recently. Garman studied the effect of shaker type and shaking angle upon fruit-stem dynamics. Through the use of high-speed photography, the motion of an artificial apple undergoing vertical and horizontal excitation was examined. Although Garman *et al.*[3] imply that resonance conditions

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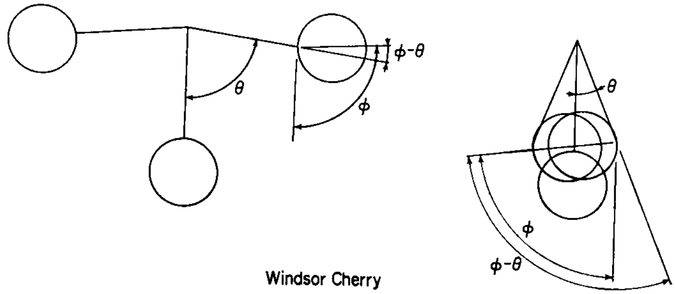


Fig. 1. The in-phase normal mode (left) and the out-of-phase mode (right) produce large bending between the support and stem and between stem and fruit, respectively

should not be utilized in fruit harvesting, they indicate that the natural frequency of apples being studied was found to range between 120 and 240 cycles/min and proceed to carry out shaking at frequencies of 300 and 420 cycles/min. This is in agreement with the results of the linear analysis[1] for resonance, namely, that the shaking frequency should be approximately twice the natural frequency.

Garman *et al.*[3] imply that fruit detachment should be achieved by large axial tensile forces produced by non-resonant motion. Note, however, that the force to detach apples,[5] oranges,[6, 7] grapefruit,[6, 7] and lemons[8] is known to decrease as much as 50% when the tensile force is substantially out of alignment with respect to the stem. The axial force required to detach the fruit may be useful in anticipating the time to harvest,[9] but it is expected to be of less value in determining how to harvest, i.e. a prediction of shaking frequency. On the other hand, vibratory fruit harvesting produces large, localized bending stresses (in contrast with a uniform stress along the stem). In agreement with this, many crops[6–8, 10–18] are known to require numerous bending cycles before detachment occurs.

In a study of macademia nuts Liang *et al.*[4] have investigated the use of multiple frequency harvesting to compensate for the variability in raceme lengths and crop maturity (and the concomitant natural frequency dependence).

To further clarify the dynamic behaviour of the fruit and stem during harvest the exceedingly useful assumption of small angles of oscillation invoked in the linear analysis[1] will now be relaxed. This, in general, is tantamount to removal of the condition of independence of the "natural frequency" and the amplitude of oscillation. More specifically, the method of Kauderer[19] will be used to find the class of initial conditions for the double pendulum which will produce periodic free vibrations of the fruit and stem. It is presumed that a better understanding of the free vibration conditions will be essential in a study of the more complex non-linear forced vibration of a multiple-degree-of-freedom system.

The practical motivation for this research is the desire to produce a unified, rational theory of the operation of vibratory harvesting equipment for a large class of crops. Consequently, calculations will be presented for representative varieties of numerous crops. The practical limitations of the mathematical analysis will be emphasized.

2. Analysis

Vibrations problems may be classed as either linear or non-linear and as either free or forced vibrations. The problem of vibratory fruit harvesting as reflected by the mathematical model of a fruit-stem system is most appropriately analysed as a non-linear forced vibrations problem. The analysis of non-linear problems is more difficult than that of linear problems and a non-linear forced system is harder to study than a non-linear free system. A non-linear analysis includes the corresponding linear analysis as a limiting case when the amplitude of vibration approaches zero, and is, therefore, useful in finding the accuracy and applicability of the simpler linear results.

In Cooke and Rand,[1] both the linear free vibrations and the linear forced vibrations of a fruit-stem system were studied. The normal modes and natural frequencies of the linear free vibrations problem were shown to play a prominent role in the linear forced problem. In this paper an analysis of the non-linear free vibrations problem will extend the previous results. It is expected that this investigation will pave the way for the eventual non-linear forced problem.

It is well-known from the classical theory of the non-linear vibrations of single-degree-of-freedom systems (Stoker[20]) that in general there is a relationship between the frequency of a vibrating system and the amplitude of that vibration. The non-linear normal mode frequency approaches that found by the corresponding linear analysis as the amplitude of oscillation approaches zero (*Fig. 2*).

Now, the system under consideration in Cooke and Rand[1] had 2 degrees of freedom and hence the linear analysis gave 2 natural frequencies. It is expected, therefore, that analagous to the results for one-degree-of-freedom systems, there will be 2 amplitude-frequency curves resulting from the non-linear analysis (*Fig. 3*). Each of these curves will correspond to a non-linear normal mode vibration. For a multi-degree-of-freedom system, Rosenberg[21] has defined a non-linear normal mode as those motions for which the co-ordinates (a) achieve zero velocity simultaneously and (b) pass through the equilibrium position simultaneously.

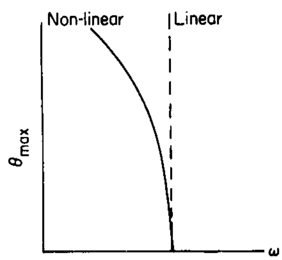


Fig. 2

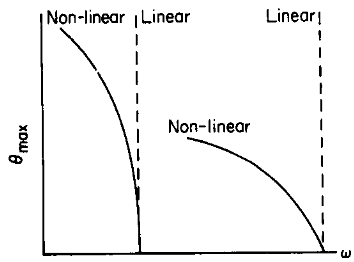


Fig. 3

Fig. 2. Amplitude-frequency relation for a single degree-of-freedom system (diagrammatical)

Fig. 3. Amplitude-frequency relation for a 2 degree-of-freedom system (diagrammatical)

Thus for a two-degree-of-freedom system there is associated with each natural frequency a normal mode shape. It is expected that the normal mode shape in a non-linear analysis will, like the frequency of vibration, also vary with the amplitude. This may be graphically represented as in *Fig. 4*. If x_1 and x_2 are generalized co-ordinates, then a normal mode is represented in the x_1–x_2 plane as a curve through the origin (and is symmetric with respect to the origin). For linear vibrations this curve is always a straight line, but for non-linear vibrations curved modal lines are generally expected.

Time appears as a parameter describing the motion of the system point along the modal line. If at $t=0$ the system starts from rest, i.e. from either end of the modal line, then the system point moves towards the origin and passes through the origin to the other end of the modal line. As the time t increases, the system point then returns to the origin and moves back to its initial rest point, the total time required being the period of the (non-linear) motion.

The variation of the non-linear normal mode with the amplitude of vibration may be graphically described by examining the rest point (end point) of a modal line (since the rest point is simultaneously a measure of the amplitude of vibration and of the geometry of the modal line) the locus of all such rest points for a particular normal mode is called a "grenzkurve" by Kauderer[19] and is shown in *Fig. 5*.

It is the purpose of the following analysis to find approximations to the non-linear normal modes for a model of a fruit-stem system. Rosenberg[21] has demonstrated that resonance of a forced non-linear system can occur in the region of a normal mode vibration.

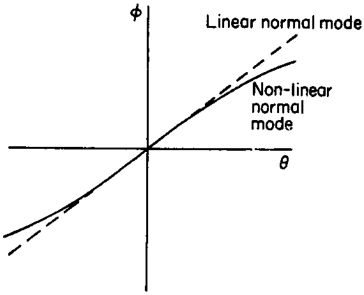


Fig. 4

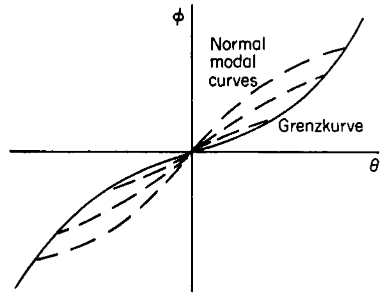


Fig. 5

Fig. 4. Normal modes in a 2 degree-of-freedom system (diagrammatical)
Fig. 5. Grenzkurve for a 2 degree-of-freedom system (diagrammatical)

Fig. 1 shows the mathematical model of a the fruit and stem. The stem is a rod of mass μ and length L. The stem stiffness is lumped in the 2 torsional springs, S and K, at the upper and lower ends of the stem, respectively. The fruit is treated as a uniform density mass M of radius R. The planar angular displacements of the stem and fruit, as measured from the vertical, are θ and ϕ, respectively, ψ is the angular displacement of the fruit about the calyx-stem end axis and has torsional spring constant C.

The kinetic energy T and potential energy V for the free vibration (Cooke and Rand[1]) are

$$T=(1/6)\,\mu\,L^2\dot{\theta}^2+(1/2)M\,[L^2\dot{\theta}^2+R^2\dot{\phi}^2+2RL\dot{\phi}\dot{\theta}\cos(\theta-\phi)]+(1/5)\,MR^2\,(\dot{\phi}^2+\dot{\psi}^2) \quad \ldots(1)$$

$$V=(1/2)\,S\theta^2+(1/2)\,K\,(\phi-\theta)^2+(1/2)\,C\,\psi^2-\mu g\,(L/2)\cos\theta-Mg\,(L\cos\theta+R\cos\phi) \quad \ldots(2)$$

Since the pendulum and tilting motions are of primary interest, we shall set $\psi\equiv 0$. Furthermore, if we introduce the following abbreviations

$b_1=(1/3)\mu\,L^2+ML^2$; $b_2=(7/5)\,MR^2$; $b_3=MRL$; $b_4=\mu(L/2)g+MgL$; $b_5=MgR$

then

$$T=(1/2)\,b_1\,\dot{\theta}^2+(1/2)\,b_2\,\dot{\phi}^2+b_3\,\dot{\theta}\,\dot{\phi}\cos(\theta-\phi) \quad \ldots(3)$$

$$V=(1/2)\,S\theta^2+(1/2)\,K\,(\phi-\theta)^2-b_4\cos\theta-b_5\cos\phi. \quad \ldots(4)$$

From Lagrange's equations the equations of motion become

$$b_1\ddot{\theta}+b_3\ddot{\phi}\cos(\theta-\phi)+b_3\dot{\phi}^2\sin(\theta-\phi)+S\theta+K\,(\theta-\phi)+b_4\sin\theta=0 \quad \ldots(5)$$

$$b_2\ddot{\phi}+b_3\ddot{\theta}\cos(\theta-\phi)-b_3\dot{\theta}^2\sin(\theta-\phi)+K\,(\phi-\theta)+b_5\sin\phi=0. \quad \ldots(6)$$

The trigonometric functions in Eqns (5) and (6) will now be expanded in a Taylor series and fourth and higher power products will be neglected. [In the linear analysis terms of second and higher order were discarded.]

$$b_1\ddot{\theta}+b_3\ddot{\phi}\,[1-(\theta-\phi)^2/2]+b_3\dot{\phi}^2\,(\theta-\phi)+S\theta+K(\theta-\phi)+b_4\left[\theta-\frac{\theta^3}{6}\right]=0 \quad \ldots(7)$$

$$b_2\ddot{\phi}+b_3\ddot{\theta}\,[1-(\theta-\phi)^2/2]-b_3\,\dot{\theta}^2(\theta-\phi)+K(\phi-\theta)+b_5\left[\phi-\frac{\phi^3}{6}\right]=0. \quad \ldots(8)$$

Kauderer's[19] approach will be utilized to obtain an appropriate solution of the form

$$\theta=A\cos\omega t \quad \ldots(9)$$

$$\phi=B\cos\omega t. \quad \ldots(10)$$

The use of Eqns (9) and (10) implies that the solution will involve straight modal lines (approximation to actual curved lines) but unlike the linear case (ϕ/θ) is a function of the amplitude (and, therefore, is also frequency-dependent). The dependence of A and B upon ω will now be developed. First Eqns (9) and (10) will be substituted into Eqns (7) and (8). Using $\cos^3 \omega t = (3/4)\cos \omega t + (1/4)\cos 3\omega t$ and neglecting $\cos 3\omega t$ terms, we obtain

$$-\omega^2 b_1 A - b_3\omega^2 B + (3/8)b_3\omega^2 B(A-B)^2 + (1/4)b_3\omega^2 B^2(A-B) + SA + K(A-B) + b_4 A - b_4 A^3/8 = 0 \quad \ldots(11)$$

$$-\omega^2 b_2 B - b_3\omega^2 A + (3/8)b_3\omega^2 A(A-B)^2 - (1/4)b_3\omega^2 A^2(A-B) + K(B-A) + b_5 B - b_5 B^3/8 = 0 \quad \ldots(12)$$

Solve Eqns (11) and (12) separately for ω^2 and equate the results.

$$\omega^2 = -\left[\frac{SA+K(A-B)+b_4A-b_4A^3/8}{-b_1A-b_3B+(3/8)b_3B(A-B)^2+(1/4)b_3B^2(A-B)}\right]$$
$$= -\left[\frac{K(B-A)+b_5B-b_5B^3/8}{-b_2B-b_3A+(3/8)b_3A(A-B)^2-(1/4)b_3A^2(A-B)}\right]. \quad \ldots(13)$$

After Kauderer, set

$$B = \lambda A \quad \ldots(14)$$

and substitute into Eqn (13) and simplify.

Obtain

$$(t_1+t_2A^2)(t_3+t_4A^2) = (t_5+t_6A^2)(t_7+t_8A^2) \quad \ldots(15)$$

where

$$t_1 = S+K(1-\lambda)+b_4;\ t_2 = -b_4/8;\ t_3 = -b_2\lambda-b_3;\ t_4 = (3\lambda-1)(\lambda-1)b_3/8;\ t_5 = K(\lambda-1)+b_5\lambda;$$
$$t_6 = -b_5\lambda^3/8;\ t_7 = -b_1-b_3\lambda;\ t_8 = \lambda b_3(\lambda-3)(\lambda-1)/8$$

or

$$aA^4+\beta A^2+\gamma = 0 \quad \ldots(16)$$

$$a = t_2t_4-t_6t_8$$
$$\beta = t_1t_4+t_2t_3-t_6t_7-t_5t_8$$
$$\gamma = t_1t_3-t_5t_7$$

Therefore,

$$A^2 = [-\beta \pm (\beta^2-4a\gamma)^{1/2}]/2a. \quad \ldots(17)$$

But for the linear case (Cooke and Rand[1]), the frequency equation is just

$$\gamma = 0. \quad \ldots(18)$$

Therefore, we desire $A^2 = 0$ (small amplitude approximation) when $\gamma = 0$. Choose the plus sign for $\beta>0$ and the minus sign for $\beta<0$ in Eqn (17). The amplitude A in Eqn (17) is a function of λ and the parameters of the model. And since Eqn (13) gives the dependence of ω upon A, B and the parameters M, R, μ, L, S, K, use Eqn (14) to obtain

$$\omega^2 = -\left[\frac{S+K(1-\lambda)+b_4-b_4A^2/8}{-b_1-b_3\lambda+[(3/8)b_3\lambda(1-\lambda)^2+(1/4)\lambda^2b_3(1-\lambda)]A^2}\right]. \quad \ldots(19)$$

We may substitute Eqn (17) into (19) and obtain ω^2 as a function of λ and the physical parameters. Furthermore, we now have relations between amplitude A and mode shape λ and between the frequency ω and the mode shape λ. Therefore, we also have an implicit relation between amplitude and frequency.

3. Numerical results and discussion

The implications of the preceding mathematical modeling will now be interpreted. When available, actual parameters from the literature are used; the other parameters are hypothetical, but are believed to be reasonable. In all cases the actual source of the data is given (see Table I.)

As already described, the important information to be gained from this analysis is contained in the amplitude-frequency relationship and the relationship between the 2 angular pendulum displacements at maximum amplitude.

An explicit graphical representation of the amplitude-frequency correspondence may be obtained in the following manner for a given set of parameters, M, R, μ, L, S and K. An assumed sequence of λ values [Eqn (14)] will be used in Eqns (17) and (19) to give the amplitude-frequency information concurrent with a mutually consistent set of A *vs* B [Eqns (9) and (10)] where $B = \lambda A$. The sequence of λ values will be bounded either above or below by the mode shape (φ/θ) obtained from the linear analysis. Lambda varies over a small range so increments of 0·001 or 0·0001 are usually suitable. Calculation is terminated if imaginary ω or A occurs in the use of Eqns (17) and (19). However, a practical limitation on the calculations arises from the approximation used in Eqns (7) and (8). Of course, higher-order approximations may be utilized, but the analytical work presented is reliable for $|\theta| \leqslant 1$ rad for the in-phase mode and $|\phi| \leqslant 1$ rad for the out-of-phase mode. In some cases the range may be extended by 40–50%, as verified by direct numerical integration of the non-linear Eqns (5) and (6) of motion.

The Hamming predictor-corrector method[22] was used to solve the general initial value problem, reformulated as a system of 4 non-linear, first-order equations. This stable, fourth-order integration procedure requires only 2 functional evaluations per integration step, after being started with a Runge–Kutta procedure. The calculation was terminated after the fifth occurrence of a value of zero for the theta displacement.

Although the limitation on the maximum angular displacement of the fruit and stem is removed, this procedure is too costly (in terms of computer time) to iteratively search for the entire amplitude-frequency curve for many sets of parameters. However, this procedure was used to verify the non-linear normal modes for several fruits. For a Golden Delicious apple with various initial conditions which correspond to a constant initial potential energy (zero kinetic energy). *Fig. 6* shows the configuration of the fruit-stem system in a θ, ϕ plane with time as an implicit parameter. The 2 curves which pass through the origin $[\theta(t) = \phi(t) = 0]$ simultaneously correspond to the non-linear normal modes—one in-phase (θ and ϕ always of the same sign) and one out-of-phase (θ and ϕ always of opposite sign). The appropriate initial conditions were found from the analysis presented above. Two other non-normal mode oscillations are shown to indicate that, in general, non-periodic motion results. The actual times elapsed in the 4 cases shown are quite different. The time-dependence for the 2 normal modes is shown in *Fig. 7*. The initial portion of the in-phase curve has been deleted for clarity of graphical presentation. The out-of-phase vibration frequency is typically much higher than the in-phase mode frequency. The out-of-phase mode departs slightly from the exact normal mode in this particular approximation; compare *Figs 6* and *7* for the simultaneity of $\phi = \theta = 0$.

Fig. 8 shows the θ, ϕ configuration curves for a Golden Delicious apple, Windsor cherry and Marsh grapefruit, respectively. (See Table I for the parameters used.) In each instance the largest initial in-phase and out-of-phase amplitude curves show borderline normal mode behaviour (based upon the approximate analytical results for initial conditions). In other words, these represent upper limits for the usefulness of the approximate analytical results presented. *Fig. 9* shows the θ—ϕ curve for the Golden Delicious apple with the larger-amplitude curve altered from that predicted by the approximate results. In this extended range, the initial ϕ for a given θ was over-estimated by the approximate results. In addition, note that this curve departs significantly from a straight line, in contrast to the corresponding linear problem. The relative bending, i.e. the ratio of bending between the fruit and stem to the bending at the upper end of the stem

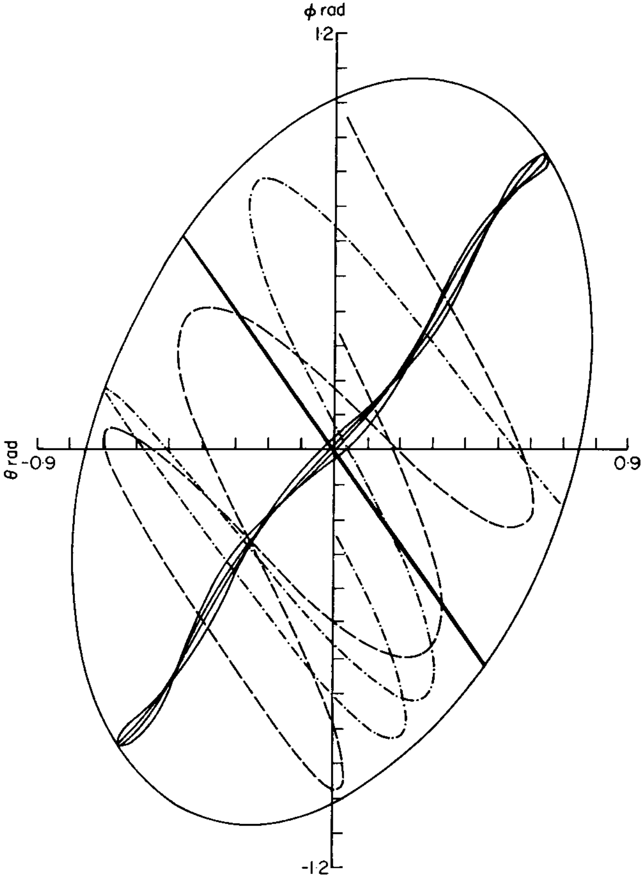


Fig. 6

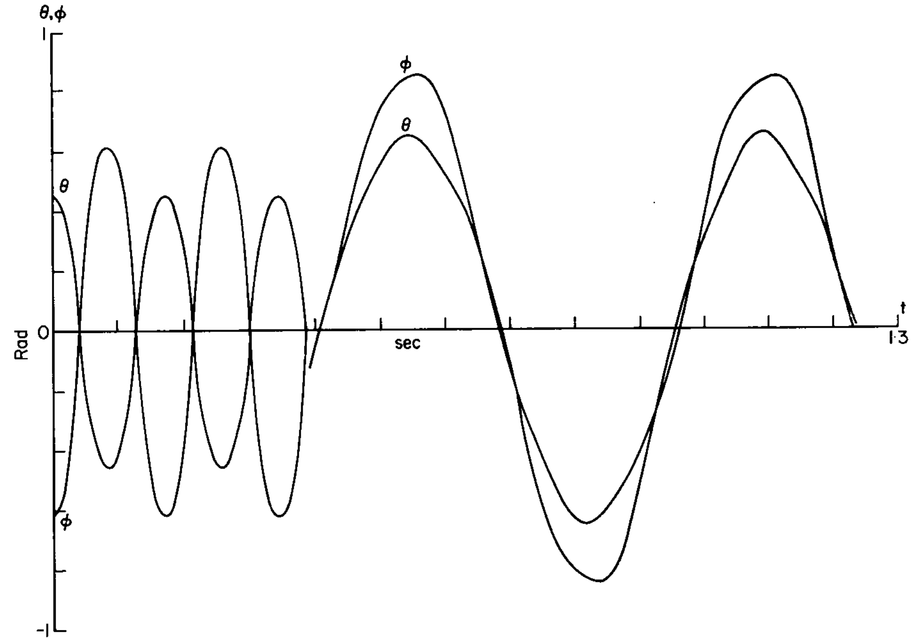


Fig. 7

Fig. 6. Configuration plane representation for a Golden Delicious apple where all initial conditions correspond to the same energy. The initial condition (0·6476, 0·8516) produces in-phase normal mode motion; (0·4572, −0·6174) produces out-of-phase motion; (0·000, 1·013) and (0·700, −0·1652) causes non-normal mode behaviour

Fig. 7. The θ and φ time-dependence for out-of-phase (left) and in-phase (right), for Golden Delicious apple. (See Table I for parameters)

TABLE I

Nonlinear free vibration frequencies and mode

	M, slug	*R, ft*	*μ, slug*	*L, ft*	*S, ft lbf/rad*	*K, ft lbf/rad*
Golden Delicious apple[a]	7·13(−3)	1·03(−1)	6·25(−6)	1·67(−1)	1·67(−2)	1·67(−2)
Windsor cherry[b]	3·64(−4)	3·52(−2)	6·73(−6)	1·21(−1)	0	0
Marsh grapefruit[c]	2·05(−2)	1·25(−1)	1·01(−4)	3·33(−1)	1·67(−2)	1·67(−2)
Montmorency cherry[d]	3·11(−4)	3·46(−2)	5·90(−6)	1·04(−1)	0	0
Redhaven peach[e]	1·60(−2)	1·27(−1)	3·00(−6)	5·00(−2)	1·67(−3)	1·67(−3)
Bartlett pear[f]	8·98(−3)	1·03(−1)	1·00(−4)	8·33(−2)	1·67(−2)	1·67(−2)
Eureka lemon[g]	6·89(−3)	9·58(−2)	1·26(−4)	4·17(−1)	1·67(−2)	1·67(−2)
Valencia orange[h]	1·71(−2)	1·25(−1)	7·58(−5)	2·50(−1)	1·67(−2)	1·67(−2)
Manzanillo olive[i]	2·39(−4)	3·42(−2)	1·00(−5)	1·01(−1)	5·00(−5)	5·00(−5)
Mature coffee[j]	1·50(−4)	2·22(−2)	1·50(−5)	2·00(−2)	5·00(−4)	5·00(−4)
Stanley plum[k]	2·42(−3)	6·63(−2)	9·47(−6)	1·67(−1)	0	0
Willamette raspberry[l]	2·47(−4)	3·12(−2)	1·00(−5)	1·01(−1)	0	5·00(−5)

$7{\cdot}13(-3) = 7{\cdot}13 \times 10^{-3}$

[a] Westwood[25]—*M, R*
[b] Tennes *et al.*[26]—*M, R, L*
[c] Rumsey[6]—*M, R, μ, L*
[d] Quackenbush *et al.*[27]—*M, R*; Studer[15]—*L*
[e] Westwood[25]—*M, R*
[f] Westwood[25]—*M, R*

*Superscripts indicate source of data

†Infinitesimal amplitude for the linear case

shapes for various fruits based on estimated parameters*

θ, rad	*ϕ, rad*	*Numerical* ω_1, *rad/sec*	*Approximate* ω_1, *rad/sec*	*Relative bending* $\left[\frac{\phi}{\theta}(\omega_1)-1\right]$	*Numerical* ω_2, *rad/sec*	*Approximate* ω_2, *rad/sec*	*Relative bending* $\left[\frac{\phi}{\theta}(\omega_2)-1\right]$
†	†		11·8	0·31		56·0	−2·38
0·65	0·85	11·5	11·5	—			
1·10	1·46	10·9	10·8	—			
1·22	1·64	10·7	10·5	—			
0·22	−0·30				48·1	47·8	—
0·67	−0·88				29·2	26·6	—
1·09	−1·20				22·7	18·9	—
†	†		14·2	0·10		54·5	−4·15
0·84	0·92	13·6	13·6	—			
1·09	1·22	13·1	13·1	—			
1·43	1·63	12·3	12·1	—			
0·07	−0·21				51·9	51·9	—
0·22	−0·73				37·4	36·0	—
0·39	−1·45				25·1	21·4	—
†	†		8·5	0·16		34·4	−3·38
0·69	0·81	8·2	8·2	—			
0·99	1·17	7·9	7·9	—			
1·34	1·61	7·5	7·4	—			
0·12	−0·28				31·3	31·2	—
0·30	−0·74				22·5	21·5	—
0·53	−1·38				15·6	13·3	—
†	†		15·1	0·11		56·0	−3·73
†	†		12·3	0·36		55·9	−1·30
†	†		13·7	0·47		72·7	−1·64
†	†		8·4	0·14		45·5	−5·04
†	†		9·4	0·21		37·7	−2·74
†	†		15·6	0·16		63·0	−3·68
†	†		44·2	0·56		328·9	−1·73
†	†		11·6	0·13		41·8	−3·24
†	†		15·5	0·09		64·9	−3·97

[g] Barnes[8]—*R, L*; Erickson and Haas[28]—*M*

[h] Coppock *et al.*[7]—*M, R*

[i] Brewer[29]—*M*; Studer[15]—*R, L*

[j] Wang[11]—*M, L*; Phillips[30]—*R*

[k] Quackenbush *et al.*[27]—*M, R*

[l] Nyborg and Coulthard[31]—*M, R, L*

Note: All other parameters are hypothetical, but are believed to be reasonable estimates.

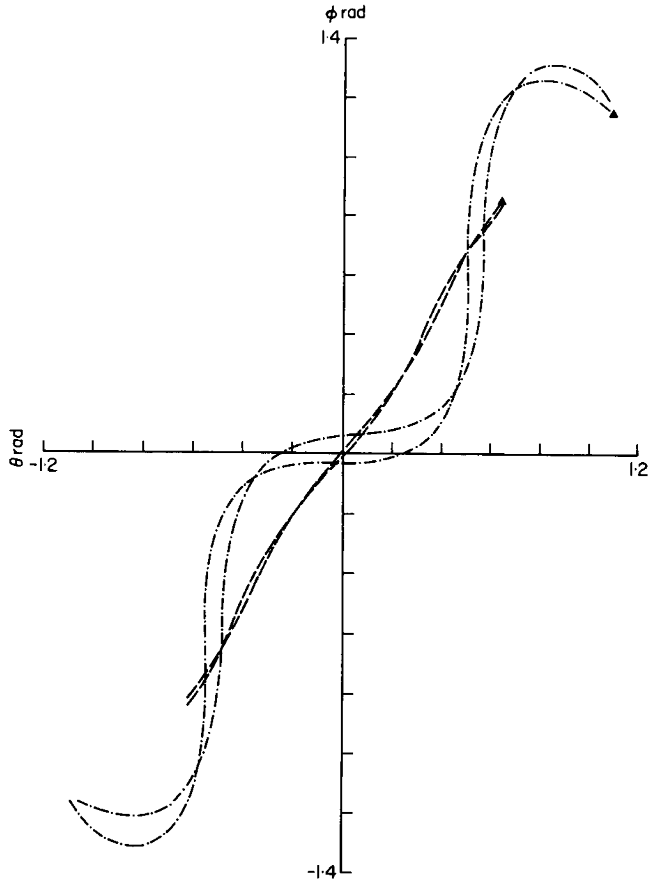


Fig. 9. The modal curve in the configuration plane departs significantly from a straight line for large amplitudes

$[(\phi/\theta)-1]$ or slope minus unity, varies throughout the cycle and is not necessarily an extremum at the endpoint.

If the endpoints of the θ, ϕ plane curves are connected for normal mode vibrations, the entire set of curves is summarized by this limiting curve or grenzkurve. The grenzkurve, then, shows the allowable initial conditions for normal mode vibration. The grenzkurve for parameters of various fruits listed in Table I are shown in *Fig. 10.* For the cases tabulated (Table I) the in-phase grenzkurve has an upward curvature (with positive slope) for Golden Delicious apple, Windsor cherry, Marsh grapefruit, Bartlett pear, Manzanillo olive, Redhaven peach, Montmorency cherry, Eureka lemon, Stanley plum, and Valencia orange; the out-of-phase grenzkurve has an upward curvature (with negative slope) for Golden Delicious apple, Bartlett pear, mature coffee, and Redhaven peach. (See *Fig. 10.*) Amplitude-frequency curves are shown in *Fig. 11* for a Golden Delicious apple, Windsor cherry, Marsh grapefruit, Montmorency cherry, Redhaven peach, Bartlett pear, Eureka lemon, Valencia orange, Manzanillo olive, coffee, Stanley plum and Willamette raspberry. The curves are indicative, rather than definitive, since some parameter estimation was involved. Calculations for several varieties (apples, 6; peaches, 7; pears, 3; cherries, 3; oranges, 3) suggest that the data presented are indicative of the characteristics of the respective crops.

The amplitude-frequency curves for a given crop retain the same general shape for the parameter changes anticipated within a mature crop. In magnitude, the amplitude-frequency curve

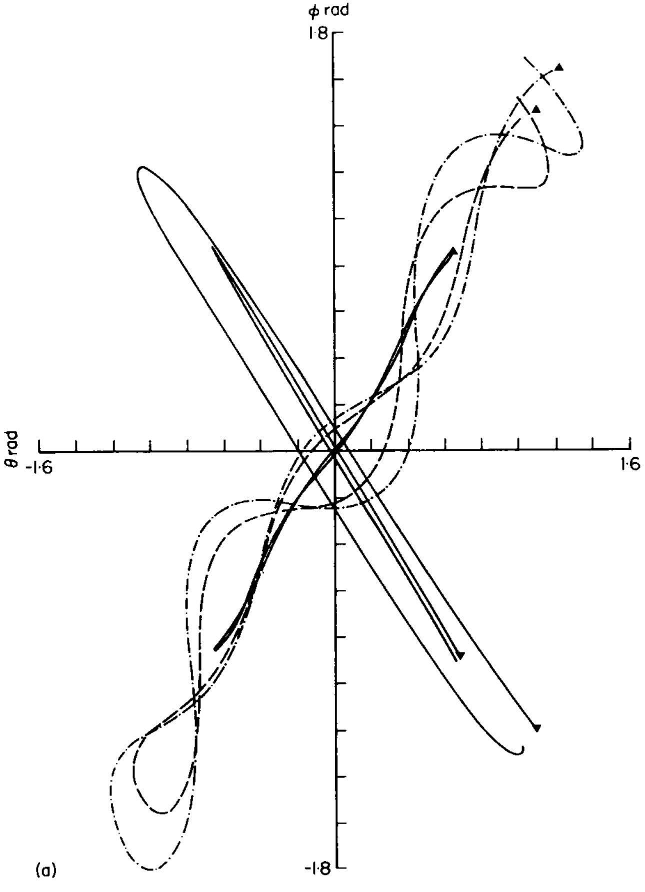
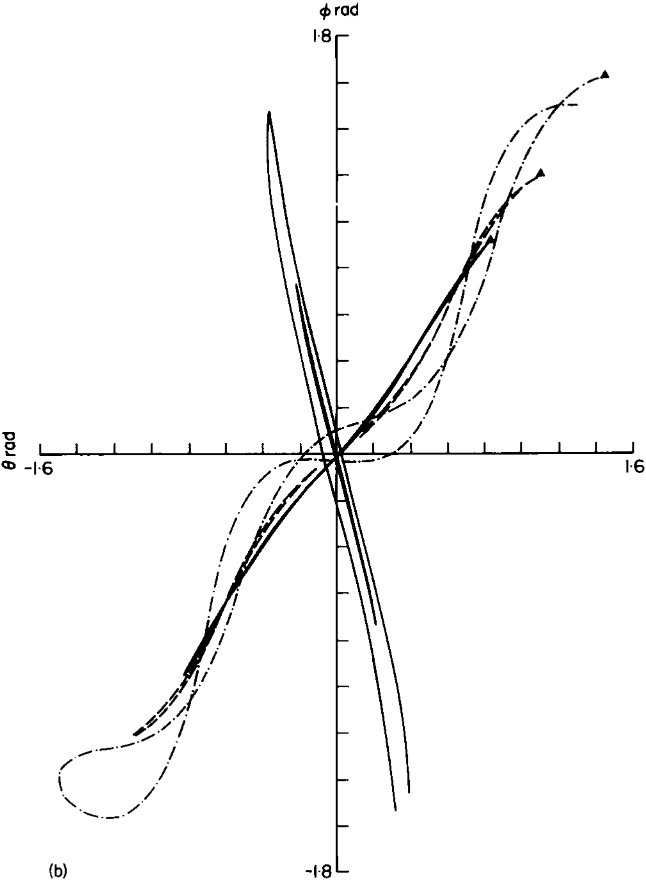
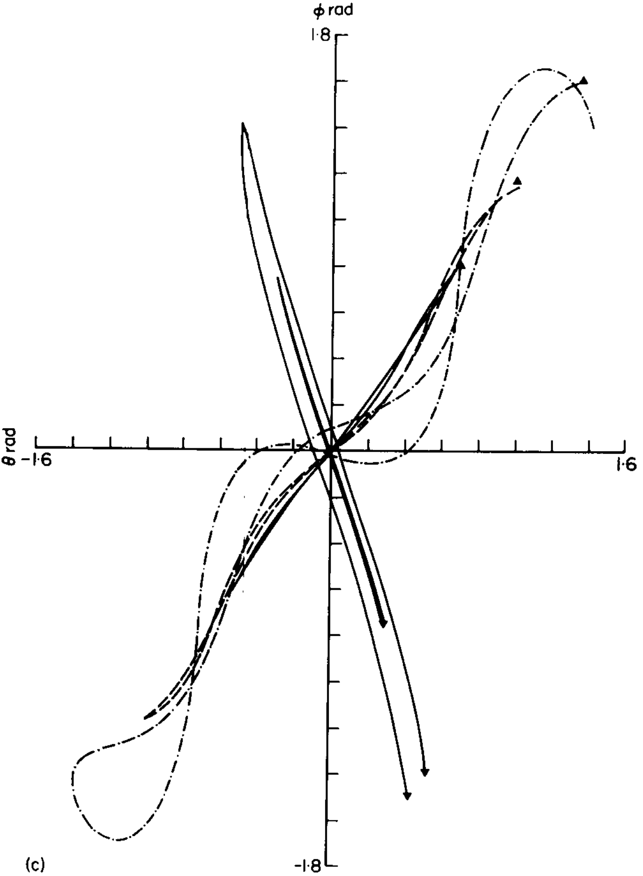


Fig. 8. Configuratio
for appr



tion plane of Golden Delicious apple (left); Windsor cherry (middle); and
pproximate normal mode motion. (See Table I for parameters and initial

6



d Marsh grapefruit (right)
l conditions)

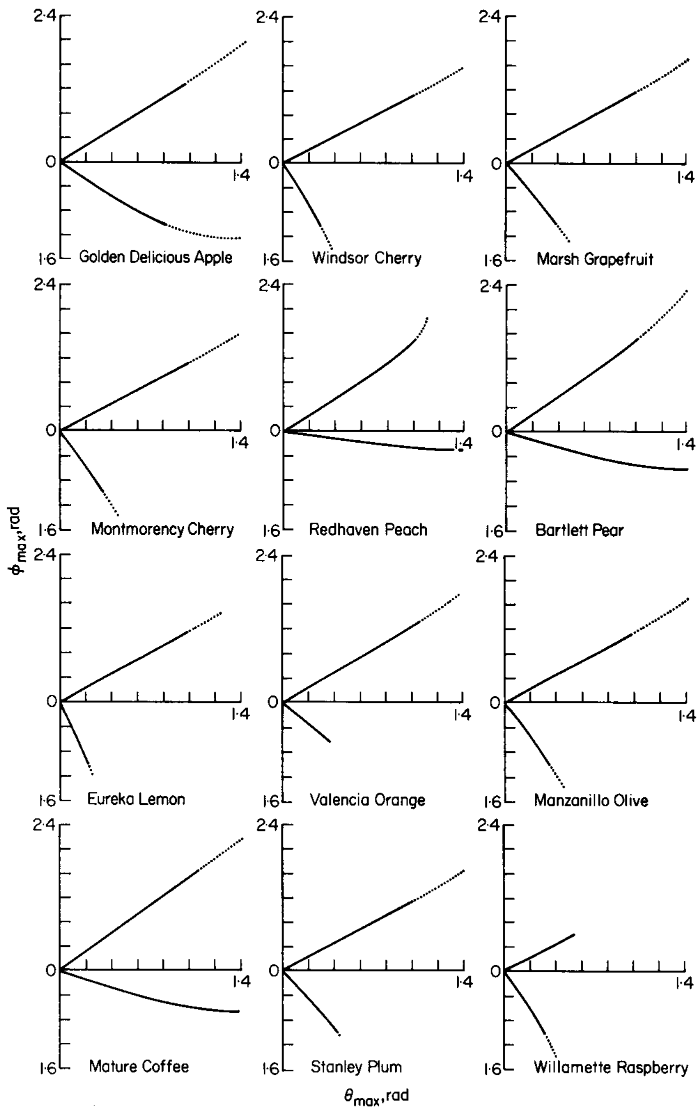


Fig. 10. The grenzkurve for each of the sets of parameters listed in Table 1 is given. All curves are symmetric with respect to the origin

for the higher-frequency (out-of-phase) mode is much more sensitive to parameter variability. This is usually expressed as a translation to lower frequencies as the stem length increases and as the spring constants decrease. The slope for the higher mode does, however, decrease slightly as the spring constants increase. The non-linear results (as with the linear results), are very insensitive to normal variations in mass (and are also insensitive to radius changes since the mass density is fairly constant).

The in-phase mode amplitude-frequency curve, in all cases, is very steep (unlike that of the higher-frequency cases), indicating that the double pendulum behaves very much as a simple pendulum. In other words, the frequency is *not* highly dependent upon amplitude in the range of displacement less than 1 rad. *Fig. 12* shows the complete elliptic integral solution[23] of the

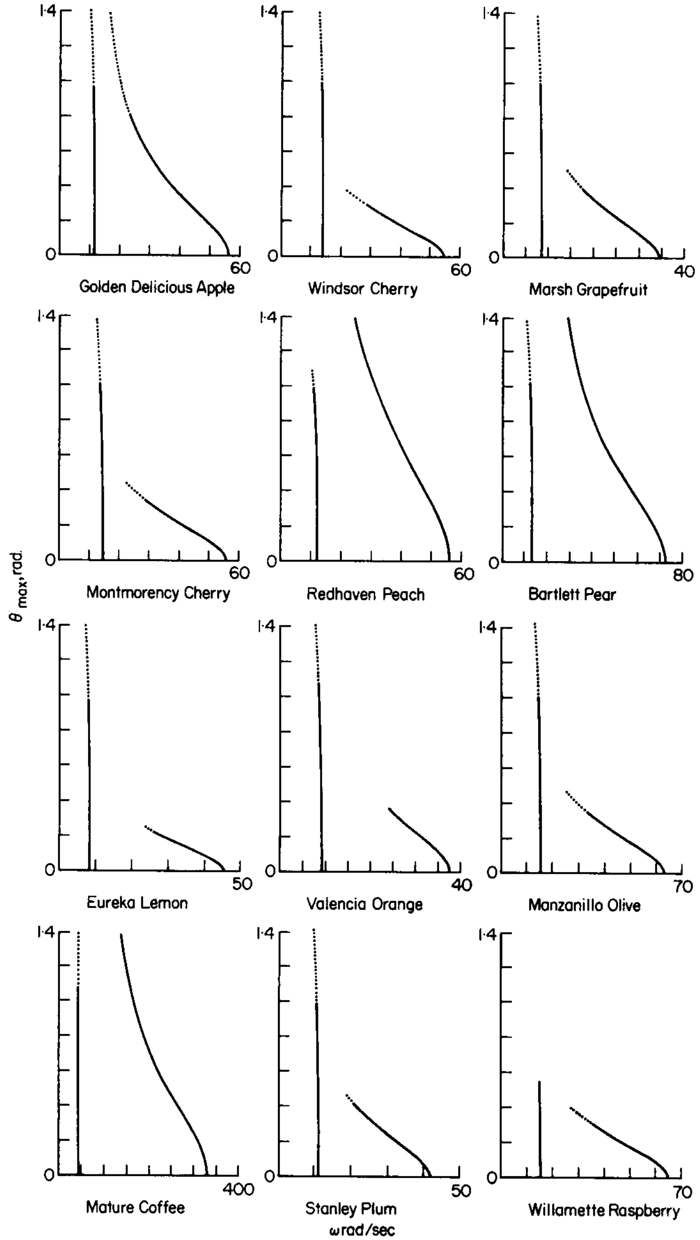


Fig. 11. The amplitude-frequency curves for the sets of parameters in Table I. (The abscissa axis has been normalized to a constant length)

amplitude-frequency curve for a simple pendulum without a spring. The curve is indeed quite similar to the curve for a Golden Delicious apple (with stiffness) treated as a simple pendulum. Specifically, the results of the numerical integration are for initial conditions of $\theta = \phi$, i.e. the fruit and stem are in alignment. The departure from normal mode behaviour is very slight.

Table II displays values for the natural frequencies and half the reported shaking frequencies. In the case of laboratory testing with horizontal excitation the frequency has not been halved.

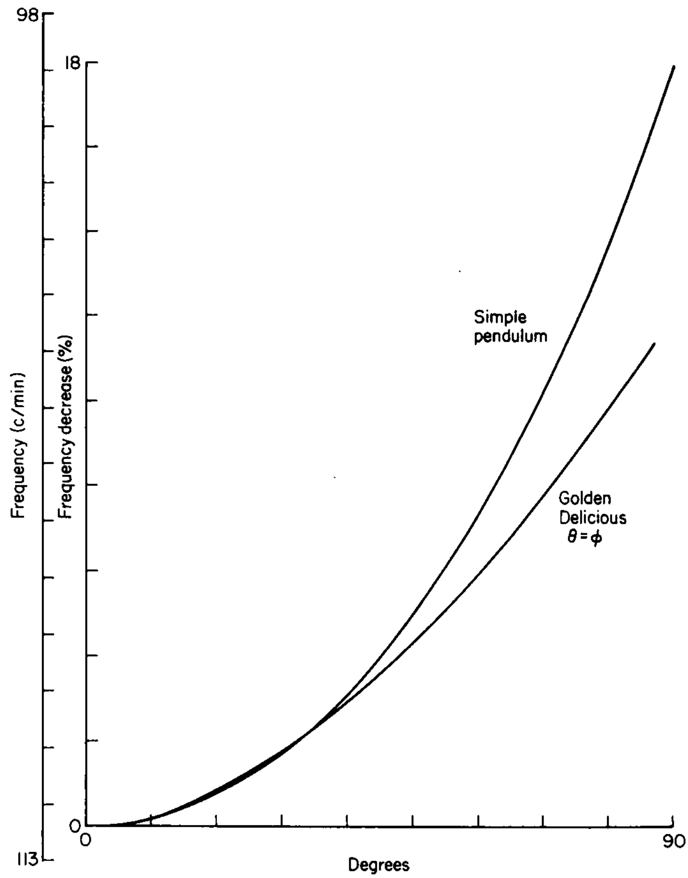


Fig. 12. The frequency as a function of initial amplitude for a simple pendulum approximates that for a Golden Delicious apple with $\theta = \phi$

Although the shaking frequencies do not necessarily represent optimal conditions, substantial agreement is evidenced between the calculated and experimental values. (A recent review of all fruit harvest mechanization research and development is now available.)[24]

4. Conclusions

For the crops for which data were available, the in-phase free vibration frequency is relatively independent of the free vibration amplitude. In fact, the non-linear effects for this mode will not be appreciable for angles less than 30° when the initial fruit and stem alignment are maintained. For angles as large as 90°, the double pendulum frequency does not depart greatly from that of an equivalent simple nonlinear pendulum; however, the frequency will have decreased with respect to the small-angle frequency by 15%.

The out-of-phase free vibration, on the other hand, is strongly affected by the amplitude of stem oscillation (*Fig. 11*). This suggests that to maintain a normal-mode free vibration condition, the frequency must drop rapidly as the amplitude increases. Although firm conclusions about the forced vibration case cannot be made from this free vibrations analysis, we conjecture that this non-linear characteristic would require that crops to be harvested without stems (remaining attached to the fruit, e.g. tart cherries) should utilize pulsating frequencies of the shaker. In this way a peak

TABLE II

Comparison of calculated natural frequency

Crop	*Calculated frequencies* cycles/min*		*One-half reported frequencies† cycles/min*		*Stem length (in)*
	pendulum	*tilting*	*pendulum*	*tilting*	
Apples	100–175	340–1000	150–205	—	—
			80–180	220–340‡	—
			200–300	—	—
			150–210	—	—
Cherries	130–160	500–850	—	525	—
			225	—	—
			—	600–900	—
Grapefruits	70–130	275–750	50	200–360‡	2·0
Peaches	120–150	500–1000	—	375–400	—
Pears	120–140	400–600	275–325	—	—
Lemons	70–90	300–500	—	550‡	4·0
Oranges	70–90	300–400	100‡	350–600‡	2·0
			—	175–200	6·0
Olives	150–250	600–1000	—	600	—
				1000	—
			225	—	1·21
Coffee	400–450	800–1200	—	1300	—
Raspberry	150–250	950–1100	—	1100	—
				300–375	—
				150–175	—

* These values are based upon estimated parameters. (See Table I)
† These reported values do not necessarily represent optimal values.
‡ The values for laboratory conditions with horizontal forcing are not divided by two.

frequency (twice the higher natural frequency) could be used to induce oscillations and the subsequent decreasing frequency would help to maintain resonance. Of course, frequency should not be reduced so low as to intrude into the range of twice the lower natural frequency, in order to minimize the percentage of stems collected. The strategy of using short bursts has, in fact, been found effective for cherries.[32] This procedure would not be expected to be as useful for crops to be harvested with stems attached, unless the parameters of biological variability, such as stem length and stiffness, are especially non-uniform within the mature crop (in which case the pulsating would serve to vibrate over a band of frequencies.) Further, the short burst technique will allow a smaller number of fruits to be harvested per burst, thus reducing the materials handling problem.

In conclusion, the double pendulum model appears to be an adequate representation for the study of the fruit-stem dynamics for many crops and, therefore, unifies a very large class of fruit harvesting work.

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and one-half the reported shaking frequency

Stroke length, (in)	*Attached stems desired*	*Laboratory conditions*	*Remarks*	*Reference*
—	yes	yes	free vibration, several varieties	Thomas[17]
—		yes	dwarf trees	Diener *et al.*[2]
1·0		yes	Jonathan	Diener *et al.*[18]
—		yes	artificial	Garman *et al.*[3]
1·0	no, yes	no	tart	Markwardt *et al.*[32]
0·125		yes	Montmorency	Studer[15]
0·4		yes	tart	Unknown[33] (Bruhn)
2·0	no	yes	Marsh	Rumsey[6]
1·0–1·5	no	no	—	Webb[34]
—	yes	no	—	Markwardt[35]
2·0	no	yes	Eureka	Barnes[8]
2·0	no	yes	Valencia	Rumsey[6]
2·25–2·75	no	no	Valencia	Brown and Schertz[36]
1·0	no	yes	Manzanillo	Lamouria and Brewer[10]
0·25		yes	Manzanillo	Lamouria and Brewer[10]
2·0		yes	Manzanillo	Studer[15]
—	no	no	—	Wang[11]; Monroe and Wang[37]
1·0	no	no	Willamette	Nyborg and Coulthard[38]
1·5		no	Red	Hughes and Ricketson[39]
4·0		no	Red	Hughes and Ricketson[39]

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Appendix

In Cooke and Rand[1] a linear analysis of the fruit-stem model studied in this paper was presented, concluding that a driving frequency equal to twice the natural frequency would produce the greatest instability of the fruit-stem system. The argument leading to Eqn (3) of that paper contains a flaw which it is the purpose of this Appendix to remedy.

In the case of no torsion springs ($S=K=0$) the linearized equations of forced motion become

$$A\ddot{x}+Bx = v \qquad \text{...(A1)}$$

where

$$x=\begin{bmatrix}\theta\\ \phi\end{bmatrix}$$

$$A = \begin{bmatrix}\left(\frac{\mu}{3}+M\right)L^2 & MLR\\ MLR & \frac{7}{5}MR^2\end{bmatrix}$$

$$B = B_0\left(1+\frac{\ddot{\eta}}{g}\right)$$

$$B_0 = \begin{bmatrix}\left(\frac{\mu}{2}+M\right)gL & 0\\ 0 & MgR\end{bmatrix}$$

$$-v = \begin{bmatrix}\left(\frac{\mu}{2}+M\right)L\ddot{\xi}\\ MR\ddot{\xi}\end{bmatrix}$$

where $\eta(t)$ is the vertical support motion (not necessarily restricted to small amplitudes) and $\xi(t)$ is the horizontal support motion. Define the co-ordinates y by

$$x = Cy \qquad \text{...(A2)}$$

where the matrix C has for its columns the time-independent eigenvectors of $A^{-1}B_0$

$$C^{-1}A^{-1}B_0\, C = D \qquad \text{...(A3)}$$

where D is a diagonal matrix having the time-independent eigenvalues of $A^{-1}B_0$ for its elements. Now since C is a constant matrix

$$\ddot{x} = C\ddot{y} \qquad \text{...(A4)}$$

and

$$AC\ddot{y}+BCy = v \qquad \text{...(A5)}$$

$$\ddot{y}+D\left(1+\frac{\ddot{\eta}}{g}\right)y = C^{-1}A^{-1}v \qquad \text{...(A6)}$$

This last equation replaces Eqn (3) of Cooke and Rand. Following the ensuing analysis presented in that paper, it is easily concluded that for the model with no torsion springs the optimal driving frequency is indeed twice the natural frequency, as conjectured.