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A MATHEMATICAL INTERPRETATION OF VIBRATORY FRUIT HARVESTING: A TUTORIAL PAPER

J. Robert Cooke
Associate Professor
Department of Agricultural Engineering
Cornell University
Ithaca, New York 14850

and

Richard H. Rand
Assistant Professor
Department of Theoretical and Applied Mechanics
Cornell University
Ithaca, New York 14850

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SUMMARY:
The dynamic behavior of tree fruit during the harvesting operation is examined mathematically. A Lagrangian formulation is used in order to study the resonant conditions and the normal modes for the forced vibration of a double physical pendulum model of the fruit-stem system. The physical insight gained from the analysis is emphasized.

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A MATHEMATICAL INTERPRETATION OF VIBRATORY FRUIT HARVESTING: A TUTORIAL PAPER[1]

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INTRODUCTION

Most economically important tree fruit crops in the United States, especially those crops which undergo subsequent processing, can now be mechanically harvested. The most widely used fruit detachment procedure involves the mechanical vibration of individual limbs or of the entire tree. An inertial shaker utilizing counter-rotating, out-of-balance masses is most frequently used to produce the vibratory motion.

During the course of development of the harvesting equipment attention was first directed to the development of the necessary hardware, then to the problem of energy transmission through the tree and finally to the behavior of the fruit and stem. The present discussion will be confined to a presentation of a theoretical framework for the interpretation of the behavior of the fruit-stem system.

There are presently two points of view on the mechanics of the detachment process.

If the fruit detachment process depends principally upon the tensile forces between the fruit and stem or between the stem and branch, then the amplitude of the vibration of the tree becomes a controlling factor. This concern then leads to a consideration of the resonant behavior of the tree which is dependent upon the geometry and structure of each tree, the damping due to leaves, the change in resonant frequency of the limbs as the fruit is detached, etc.

On the other hand, if bending motions in the stem significantly enhance the detachment process, attention should be directed to the conditions which promote such bending. The stem undergoes a localized decrease in mechanical strength with approaching maturity and the formation of an abscission layer. The question then becomes one of exciting bending motions at that point. Citrus fruits are normally harvested without stems attached to reduce punctures during handling and to improve product appearance, while apples are normally harvested with stems intact to reduce spoilage. Some crops such as cherries may be

1. This paper was prepared as a contribution to the ASAE A-212 Committee's tutorial series of papers on applications of mathematics.

harvested with or without stems depending upon the variety and intended use of the product.

The vibratory response of any physical system, be it a child in a swing set, the worn front tire on an old and worn-out car, or the fruit-stem system, responds more vigorously to certain excitation frequencies. We shall show that this does indeed apply to the vibration of fruits. In addition, we shall describe the bending motions which are associated with these special (resonant) frequencies for the fruit and stem. In particular, we shall describe conditions which will produce bending at the abscission layer.

Formulation of the Model

The rather extensive literature on mechanical fruit harvesting has been reviewed by Cooke and Rand (1969), Rand and Cooke (1970), and Parchomchuk and Cooke (1972) and will not be repeated here. This tutorial paper is based upon the studies set forth in these three papers.

The fruit-stem system will be modeled as the double physical pendulum shown in Figure 1. We shall assume that the fruit and stem may vibrate without impacting other fruit or the tree structure. The fruit and stem are assumed to have a preferred equilibrium configuration. The stiffness of the stem is incorporated into the model through springs which resist bending motions. Although dissipation effects could be incorporated, this will be left as an exercise for the student since the resonant frequency is not greatly reduced by its inclusion.

The excitation of the model will be assumed to consist of a periodic horizontal movement of the upper end of the stem. To appreciably reduce the mathematical complexity, the vertical component of excitation will be neglected. (See the exercises for a discussion of this extension of the model.) Only planar motion will be considered. Finally, the excitation <u>frequency</u> at the stem is assumed to be equal to the shaker frequency, a condition which is usually reached very quickly for most trees.

The double pendulum model proposed is believed to be sufficiently general as to allow predictions for a variety of crops such as apples, cherries, coffee, grapefruit, lemons, olives, oranges, plums, raspberries, etc. Sample calculations have been given in the referenced papers.

In order to establish the conditions for resonance (i.e., dynamic instability) which accentuate localized bending in the stem, the equations of motion for the fruit and stem must be established. Lagrange's equations will be used to formulate the equations of motion in a very direct manner. The equations may be written in a very methodical manner using equation 1 once the kinetic and potential energy expressions have been written with respect to an inertial frame of reference.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0, \; i = 1, 2 \qquad (1)$$

where $q_1 = \theta$ and $q_2 = \phi$ are the generalized coordinates chosen to uniquely describe the location of the stem and fruit with respect to the vertical.

$$L = L(q_i, \dot{q}_i) = T - V \qquad (2)$$

where T and V are the kinetic and potential energies, respectively.

For each rigid body the kinetic energy can be written as the sum of two terms: a) the kinetic energy T_{cm} of the mass treated as a particle at the center of mass of the body and b) the rotational kinetic energy T_{rot} about its center of mass, regardless of the actual instantaneous center of rotation.

$$T = T_{cm} + T_{rot} \qquad (3)$$

If (x_1, y_1) and (x_2, y_2) are the coordinates in an inertial frame of reference (Figure 1) of the centers of mass for the rigid bodies representing the stem and fruit of mass μ and mass M, respectively, then the kinetic energy T_{cm} is

$$T_{cm} = (1/2)\mu\,(\dot{x}_1^{\,2} + \dot{y}_1^{\,2}) + (1/2)M\,(\dot{x}_2^{\,2} + \dot{y}_2^{\,2}) \qquad (4)$$

From Figure 1

$$\begin{aligned} x_1 &= \xi + D \sin\theta \\ y_1 &= D \cos\theta \\ x_2 &= \xi + L \sin\theta + R \sin\phi \\ y_2 &= L \cos\theta + R \cos\phi \end{aligned} \qquad (5)$$

where $\xi(t)$ is the horizontal excitation; L, stem length; and D, and R the center of mass location of each body with respect to the pivot point of each.

The components of velocity may be obtained by differentiation with respect to time where ξ, θ and ϕ are time dependent. Therefore,

$$T_{cm} = (1/2)\mu[D^2 \dot{\theta}^2 + 2D \dot{\xi} \dot{\theta} \cos \theta]$$

$$+ (1/2)M [L^2 \dot{\theta}^2 + R^2 \dot{\phi}^2 + 2L \dot{\xi} \dot{\theta} \cos \theta$$

$$+ 2R \dot{\xi} \dot{\phi} \cos \phi + 2R L \dot{\theta} \dot{\phi} \cos (\theta-\phi)] \qquad (6)$$

The rotational kinetic energy T_{rot} becomes

$$T_{rot} = (1/2)I_1 \dot{\theta}^2 + (1/2)I_2 \dot{\phi}^2 \qquad (7)$$

where I_i is the moment of inertia of each body about its own center of mass.

The potential energy V depends upon the deformation of the springs with rotational constants S and K and upon the vertical motion of the masses, where the acceleration of gravity is g.

$$V = (1/2)S\theta^2 + (1/2)K (\theta-\phi)^2 - \mu gD \cos \theta$$

$$- Mg (R \cos \phi + L \cos \theta) \qquad (8)$$

The Lagrangian becomes

$$L = T_{cm} + T_{rot} - V \qquad (9)$$

using equations 6, 7 and 8. Substituting 9 into equation 1 with $q_1 = \theta$ gives

$$(\mu D + ML) \ddot{\xi} \cos \theta + (\mu D^2 + ML^2 + I_1) \ddot{\theta}$$

$$+ MDL [\ddot{\phi} \cos (\theta-\phi) + \dot{\phi}^2 \sin (\theta-\phi)]$$

$$+ K\theta + S(\theta-\phi) + (\mu gD + MgL) \sin \theta = 0 \qquad (10)$$

and for $q_2 = \phi$ gives

$$(MR^2 + I_2)\ddot{\phi} + MR\ddot{\xi}\cos\phi$$

$$+ MRL\ [\ddot{\theta}\cos(\theta-\phi) + \dot{\theta}^2\sin(\theta-\phi)]$$

$$+ S(\phi-\theta) + MgR\sin\phi = 0 \qquad (11)$$

These equations are nonlinear and therefore difficult to treat. For an approximate study of the system's behavior, it is reasonable to linearize these equations if attention is restricted to small θ and ϕ. If the sine and cosine terms of their Taylor series expansions, i.e., $\sin\theta\approx\theta$ and $\cos\theta\approx 1$ are used, and the remaining two nonlinear terms assumed to be negligible, then the equations of motion become

$$(\mu D^2 + ML^2 + I_1)\ddot{\theta} + MRL\ddot{\phi}$$

$$+ (K+S)\theta - S\phi + (\mu gD + MgL)\theta = -\ddot{\xi}(\mu D + ML) \qquad (12)$$

and

$$(MR^2 + I_2)\ddot{\phi} + MRL\ddot{\theta} + S(\phi-\theta)$$

$$+ MgR\phi = -MR\ddot{\xi} \qquad (13)$$

These last equations may be written in the concise matrix form

$$\underline{M}\,\ddot{\underline{X}} + \underline{K}\,\underline{X} = \underline{f}(t) \qquad (14)$$

where

$$\underline{X} = \begin{bmatrix} \theta \\ \phi \end{bmatrix} \qquad (15)$$

$$\underline{M} = \begin{bmatrix} \mu D^2 + ML^2 + I_1 & MRL \\ MRL & MR^2 + I_2 \end{bmatrix} \qquad (16)$$

$$\underline{K} = \begin{bmatrix} K + S + \mu gD + MgL & -S \\ -S & S + MgR \end{bmatrix} \quad (17)$$

$$\underline{f} = \begin{bmatrix} -(\mu D + ML)\ \ddot{\xi} \\ -MR\ \ddot{\xi} \end{bmatrix} \quad (18)$$

If $\xi = A \cos \Omega t$ (19)

then

$$\underline{f} = \underline{C} \cos \Omega t \quad (20)$$

with

$$\underline{C} = \begin{bmatrix} A\Omega^2\ (\mu D + ML) \\ A\Omega^2\ MR \end{bmatrix} \quad (21)$$

Using equation 20, equation 14 becomes

$$\underline{M}\ \ddot{\underline{X}} + \underline{K}\ \underline{X} = \underline{C} \cos \Omega t \quad (22)$$

Equation 22 may be written in simpler form if premultiplied(on the left) by the inverse of $\underline{M}$

$$\ddot{\underline{X}} + \underline{M}^{-1}\ \underline{K}\ \underline{X} = \underline{M}^{-1}\ \underline{C} \cos \Omega t. \quad (23)$$

Equation 23 is then expressed in terms of a new dependent variable $\underline{y}$ related to the original by

$$\underline{X} = \underline{R}\ \underline{y} \quad (24)$$

where $\underline{R}$ is a 2 x 2 matrix (to be found.) $\underline{R}$ is to be chosen such that the two equations represented by (23) may be solved independently after being transformed by (24):

$$\underline{R}\,\ddot{\underline{y}} + \underline{M}^{-1}\,\underline{K}\,\underline{R}\,\underline{y} = \underline{M}^{-1}\,\underline{C}\cos\Omega t \tag{25}$$

Multiply on the left by $\underline{R}^{-1}$ to obtain

$$\ddot{\underline{y}} + \underline{R}^{-1}\,\underline{M}^{-1}\,\underline{K}\,\underline{R}\,\underline{y} = \underline{R}^{-1}\,\underline{M}^{-1}\,\underline{C}\cos\Omega t \tag{26}$$

If $\underline{R}^{-1}\,\underline{M}^{-1}\,\underline{K}\,\underline{R} = \underline{D}$ is a diagonal matrix, then the equations have the simpler form

$$\ddot{\underline{y}} + \underline{D}\,\underline{y} = \underline{B}\cos\Omega t \tag{27}$$

$$\text{where } \underline{B} = \underline{R}^{-1}\,\underline{M}^{-1}\,\underline{C} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \quad \text{and} \tag{28}$$

$$\underline{D} = \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} \tag{29}$$

or in scalar form,

$$\ddot{y}_1 + \omega_1^2 y_1 = B_1\cos\Omega t \tag{30}$$

$$\ddot{y}_2 + \omega_2^2 y_2 = B_2\cos\Omega t \tag{31}$$

In order to accomplish this, choose the columns of $\underline{R}$ to be two linearly independent eigenvectors of the matrix $\underline{M}^{-1}\underline{K}$, when the diagonal elements ω_i^2 of $\underline{D}$ will be the eigenvalues of $\underline{M}^{-1}\underline{K}$ (Tong, 1960, p. 180+).

That is, if

$$\underline{R} = \begin{bmatrix} 1 & 1 \\ c_1 & c_2 \end{bmatrix} = \begin{bmatrix} r_1 , r_2 \end{bmatrix} \tag{32}$$

$$\underline{r}_i = \begin{bmatrix} 1 \\ C_i \end{bmatrix} \quad (33)$$

then the column vectors $\underline{r}_i$ satisfy the equation

$$\underline{M}^{-1} \underline{K} \underline{r}_i = \omega_i^2 \underline{r}_i \quad (34)$$

or

$$\underline{K} \underline{r}_i = \omega_i^2 \underline{M} \underline{r}_i \quad (35)$$

For a nontrivial solution to (35), the determinant must vanish (Tong, 1960, p. 175-183):

$$\det (\underline{K} - \omega_i^2 \underline{M}) = 0 \quad (36)$$

Equation (36) is a quadratic on ω_i^2. Corresponding to each root (ω_1^2 and ω_2^2), equation (35) will give a value of C (C_1 and C_2) and an eigenvector ($\underline{r}_1$ and $\underline{r}_2$).

Now consider the resulting differential equations (30) and (31).

We desire the largest displacements (and stresses) for fruit removal. Equation (30) will have maximum response if $\Omega = \omega_1$; when the steady state solutions (independent of initial conditions) are

$$y_1 = [B_1 t \sin \omega_1 t]/[2\omega_1] \quad (37)$$

$$y_2 = B_2 [\cos \omega_1 t - \cos \omega_2 t]/[\omega_2^2 - \omega_1^2] \quad (38)$$

y_1 in equation (37) grows in time and represents the physical process of resonance. Similarly y_2 can be made to grow in time while y_1 stays bounded by choosing $\Omega = \omega_2$. For values of Ω other than ω_1 or ω_2, both y_1 and y_2 remain bounded.

We may use $\underline{R}$ to obtain the solution for the original independent variables θ and ϕ.

$$\theta = y_1 + y_2$$

$$\phi = C_1 y_1 + C_2 y_2 \qquad (39)$$

If $\Omega \simeq \omega_i$, then for large t, $y_i >> y_j$, $(i \neq j)$ so

$$\theta \simeq y_i$$

$$\phi \simeq C_i y_i$$

or

$$\phi \simeq C_i \theta \qquad (40)$$

Discussion of Results

From equation 40 we may conclude that for steady state motion at $\Omega \simeq \omega_i$ the angular displacement of the fruit from the vertical is proportional to the angular displacement of the stem from the vertical. Furthermore, θ and ϕ will simultaneously pass through the equilibrium configuration. This periodic behavior constitutes normal mode motion.

If C_i is positive, the fruit and stem will be in-phase, i.e., θ and ϕ will have the same sign; but if C_i is negative, the fruit and stem are out-of-phase. These two types of motion are depicted in Figure 2. The ratio of bending of the lower pivot to the bending of the upper pivot is given by the bending ratio

$$(\phi-\theta)/\theta = (\phi/\theta) - 1 \qquad (41)$$

If the absolute value of this expression is less than unity, the bending is greater at the upper end than at the lower end. The converse is true if the absolute value is greater than unity. Based upon reasonable estimates of the parameters for apples, the bending ratio is in the approximate range of 0.1 to 0.5 for the lower root of equation 36 and between -1.4 and -2.0 (approximately) for the higher frequency. This means that at the lower frequency ω_1 the fruit and stem are in-phase and that bending between the stem and branch is promoted. Therefore, if localized bending of the stem appreciably enhances detachment, there exists a bias for the fruit to be harvested with the stem still attached to the fruit. On the other hand, the higher frequency will increase the probability of separation without the stem.

An additional implication of this observation is that the resonant frequency of the desired mode should be used as the shaker frequency. After having established the frequency range for the desired mode, the shaker stroke length can then be increased (at the specified frequency) until the rate of detachment is acceptable for the specified crop, age of orchard, etc. If the shaker stroke length is too small, the duration of the resonant condition might be excessively long, leading to bruising; but if it is too large, the damage to the tree will be unacceptable. Tree motion is kept as small as possible using this strategy. [The widely used inertia-type shaker has historically included means to regulate the frequency, not the stroke length.]

The stem length, as is true for a simple one degree-of-freedom pendulum, is an important factor influencing the resonant frequency while the mass is relatively unimportant. The spring constants also can appreciably affect the frequency.

The existence of an abscission layer at one or both ends of the stem can also influence the detachment process. Specifically, if one end of the stem is much weaker than the other, separation could occur before the normal mode is established. The possibility of this condition would increase as maturity approached and passed.

Many fruits can be approximated as a sphere for purposes of assigning a moment of inertia about the center of mass. The lower pivot point for apples, however, should be taken to be in the stem cavity or dimple which is within the region of the equivalent sphere.

For certain frequencies the small oscillations are stable and will remain small. But for certain other frequencies we have shown that the deflections will cease to be small. Although not demonstrated above for large angles, the unbounded motion predicted by the linear analysis will, in general, lead to large deflections(and fruit removal).

Conclusions

The double pendulum model appears to be a suitable model for the unification of the experimental harvesting studies on numerous crops. For crops in which the bending of the stem appreciably enhances fruit detachment, a frequency dependency is predicted. The possibility of localizing the bending to harvest fruit with or without stems, as desired, is described. Therefore, experimental studies might be made more efficiently if the frequency (or frequency range) is established first.

The shaker stroke length might then be increased from a small value until the harvesting conditions are satisfied.

Generalizations

Damping was not included in the model presented here. The effect of damping will be to slightly reduce the predicted resonant frequency (Parchomchuk and Cooke, 1972). A linearized analysis which includes a vertical component of excitation reveals that resonance will also occur at excitation frequencies twice the values predicted above. This has led to the prediction that lower frequencies than are being used for apples and higher frequencies than are being used for tart cherries should be examined (Cooke and Rand, 1969).

Consideration of the non-linear behavior of the model led to knowledge that the in-phase mode free vibration frequency is relatively insensitive to the angle of oscillation while the out-of-phase mode is very sensitive to the amplitude. This led to a suggestion that a pulsating frequency be used to harvest crops if the stems are not to remain attached to the fruit.

Exercises

The reader is encouraged to test his comprehension of the fruit-stem dynamics by studying the following exercises. Some of the questions require only a more detailed understanding of the theory presented, but others lead to an extension of the model.

1. Conduct a review of the literature on the theories of stem failure. Should the criterion be maximum tensile force, maximum bending torques, fatigue, etc.?
2. What is an inertial frame of reference?
3. Rederive the equations of motion using Newton's equations. The equations using both approaches must be equivalent. Why is I_i taken about the center of mass rather than about the pivot point in the equations of motion?
4. Using a Rayleigh's dissipation function, derive the equations of motion which include damping due to the bending in the stem. (See Parchomchuk and Cooke, 1972).
5. a) Show that the equations of motion for the <u>free</u> vibration of the double pendulum with damping can be linearized to the form:

$$a_{11}\ddot{q}_1 + b_{11}\ddot{q}_1 + c_{11}q_1 + a_{12}\ddot{q}_2 + b_{12}\dot{q}_2 + c_{12}q_2 = 0$$

$$a_{12}\ddot{q}_1 + b_{12}\dot{q}_1 + c_{12}q_1 + a_{22}\ddot{q}_2 + b_{22}\dot{q}_2 + c_{22}q_2 = 0$$

   b) Find expressions for the coefficients.
6. How is the free vibration behavior of a linear system related to the forced vibration behavior?
7. Show that the frequency equation for exercise 5 is a biquadratic (quartic) polynomial.
8. Write a computer program to find the roots of the equation from exercise 7. Consider the Lin-Bairstow iterative method. (See McCalla, T. R., 1967. Introduction to Numerical Methods and Fortran Programming, p. 115 or Ralston, Anthony and Herbert Wilf, 1967. Mathematical Methods for Digital Computers, Vol. II, chapter 2).

9. Describe the matrix formulation for the vibration problem of exercise 5. (See Tong, K. N, 1960. Theory of Mechanical Vibration, p. 188+; Pipes, L. A., 1963. Matrix Methods for Engineering, p. 254+; Fraser, R. A., W. J. Duncan, A. R. Collar, 1960. Elementary Matrices and Some Applications to Dynamics and Differential Equations. Cambridge University Press, chapters 9 and 10).

10. Formulate the equations of motion when a vertical component of excitation exists.

11. Show that if S=K=0, in exercise 10, a resonance occurs at twice the excitation frequency of the value found in the analysis with purely horizontal excitation.

12. Using the Mathieu stability chart, rationalize the prediction that as the vertical excitation amplitude increases, the frequency range for resonance is increased.

13. Propose a laboratory technique for measuring the parameters used in the model.

14. Comment upon the feasibility of using the natural (or chemically induced) variations in the parameters to aid in selective harvesting of mature fruit (leaving the immature fruit for subsequent harvest).

15. Solve the forced, nonlinear equations of motion using
    a) a digital computer with a numerical integration scheme such as the Runge-Kutta or Hamming Predictor-Corrector methods (See Conte, S. D. and Carl deBoor, 1972. Elementary Numerical Analysis, chapter 6).
    b) an analog computer or a digital simulation of an analog (e.g., CSMP). Note that many multipliers are required.

16. Modify the equations of motion to be an inverted double pendulum to model blueberries. Use data presented by Soule to compute the harvesting frequencies if the fruit is to be harvested without short stems attached. (See Soule, Hayden, Jr., 1970. "Investigation of some aerodynamic properties of lowbush berries", Trans. of ASAE 13(1): 114.)

17. Show that if nonlinear effects are included, there exists a dependency of the free vibration frequency upon the angle of oscillation. (See Rand and Cooke, 1970.)

18. Improve upon the approximate nonlinear analysis of Rand and Cooke by assuming that the free vibration is better presented by

$$\theta = A_1(\omega) \cos \omega t + A_2(\omega) \cos 3\omega t$$

$$\phi = B_1(\omega) \cos \omega t + B_2(\omega) \cos 3\omega t$$

Find A_1, A_2, B_1 and B_2 as a function of frequency ω. Rather than seeking a noniterative solution, use the Rand and Cooke results as a first approximation and devise a numerical scheme to evaluate A_1, A_2, B_1 and B_2. (See Kauderer, Hans, 1958. Nonlinear Mechanics, p. 603. Note: Text in German).

19. Compare the approach suggested in exercise 18 with the geometrical approach of G. Nadig, "Geometrical Dynamics of Certain Nonlinear Conservative Systems with Two Degrees of Freedom", Ph.D. Thesis, Cornell University, 1972.

20. Formulate the fruit stem system as a double spherical pendulum (to remove the restriction to planar motion). See Symon, Keith R., 1960. Mechanics, p. 384.

21. Review the papers of Stafford, J. R. and R. G. Diener (ASAE Paper 71-645) "Instability and Nonlinear Response of a Fruit Stem with Regard to Fruit Detachment" and (ASAE Paper 72-649) "Design Criteria for Minimizing Predetachment Fruit Damage During Mechanical Shaking". The stem is assumed to be elastic and the conditions required to produce buckling are examined.

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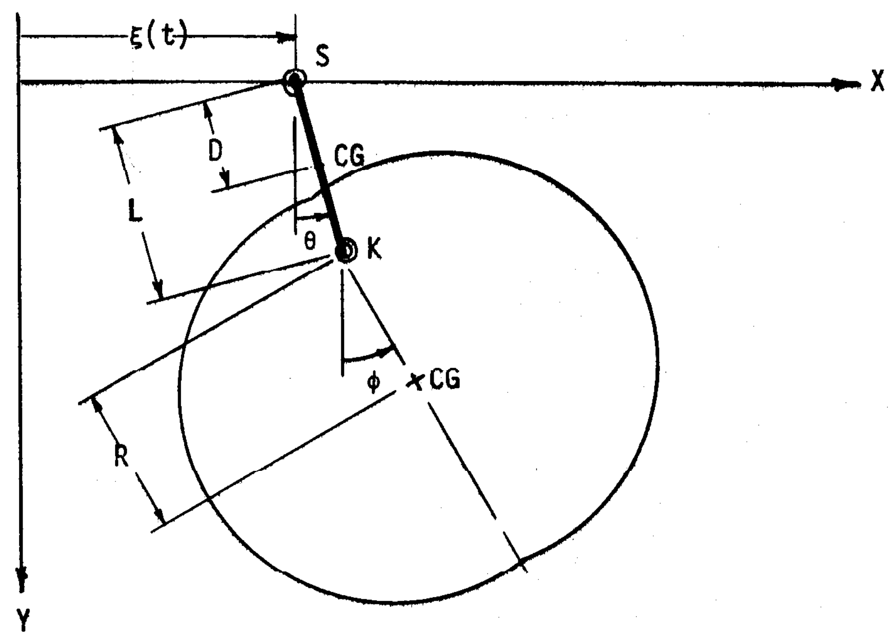

Figure 1. Double Pendulum Model of Fruit-stem system

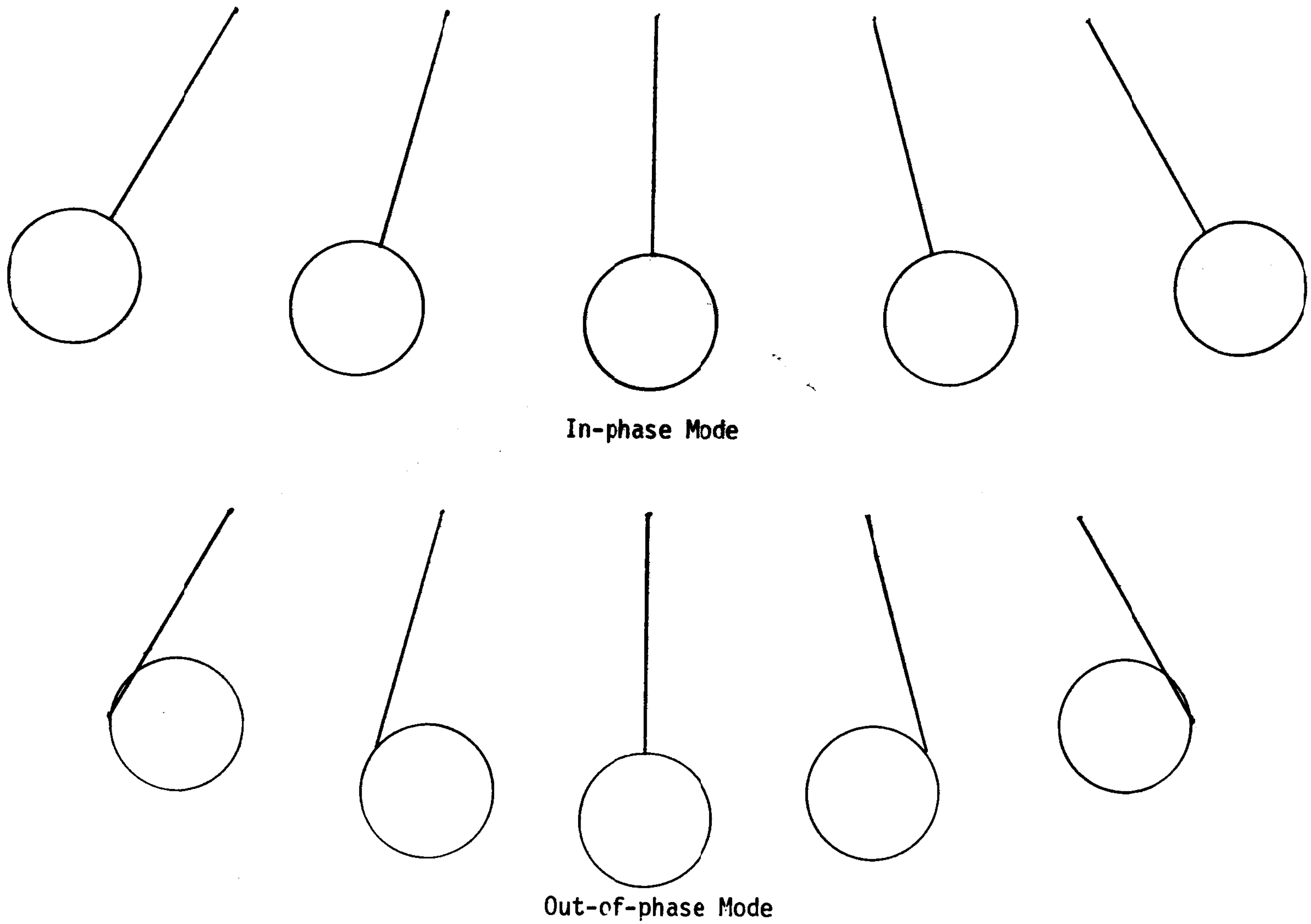

Figure 2. Normal Mode Behavior. The relative bending is greater at the upper and lower pivots for in-phase and out-of-phase motion respectively.