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The Dynamics of Resonance Capture

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ABSTRACT The concept of resonance in dynamics may be described as follows: Two distinct dynamical components (such as modes or drivers) are said to be in resonance if their associated frequencies are commensurate, that is, if their ratio is a rational number. The most important case is certainly 1:1 resonance, although many important examples involve 2:1 resonance, as well as other ratios involving small integers. The importance of resonance is that it permits energy to travel efficiently between the components that are in resonance. The most famous example is of course the destructive effects caused by forcing an elastic system at a frequency close to its natural frequency. This lecture is concerned with another resonance phenomenon called capture into resonance.

Capture into resonance may be illustrated rather than defined by the following three examples:

1. Imagine a slightly unbalanced wheel attached to an elastic support. If a constant torque is applied to the wheel, then we would expect that the wheel would begin to spin faster and faster. A surprising thing happens, however, when the wheel's angular speed gets close to the natural frequency of the elastic support. The wheel may cease to spin up, and the energy of the applied torque may instead produce large motions of the support. The wheel is said to be captured into resonance. [17],[24],[23],[25],[18].

2. Imagine a pendulum which is rotating (going over the top) in a direction opposite to an applied torque. As time goes by, the torque reduces the angular velocity of the pendulum and eventually reverses the direction of the motion. We would expect that the pendulum would eventually begin to rotate over the top in the direction of the applied torque. This is indeed the case if the torque were a constant. However, if the magnitude of the torque smoothly changes from one value to another (maintaining the same direction), the pendulum may cease to spin up, and may instead end up undergoing small oscillations about a non-vertical equilibrium position (in which the moment of the applied torque and that of gravity cancel each

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other.) [22].

3. It is well known that as the Moon travels in its orbit about the Earth, it always keeps one face pointing towards the Earth. How might this situation have arisen? In order that the Moon keep one side facing the Earth, it must complete one revolution about its axis in one orbital period. That is, this motion involves a 1:1 resonance between the motion of the Moon as a particle in orbit, and its motion as a rotating solid body. Imagine that at some time in its history the Moon did not have one side always facing the Earth. A model of the Moon's motion shows that tidal friction could provide a torque which gradually changed the Moon's rotation rate until it was captured into its present 1:1 resonance. [9],[21].

Many other examples exist [8],[6], including applications to dual-spin spacecraft [17],[16],[14],[29],[18],[19],[15],[12].

In all these examples the mathematical model reveals that there is a slowly moving separatrix which surrounds a center. *Captured motions correspond to those orbits which cross the separatrix and end up circulating around the center.* Some motions do not enter the separated region and are not captured. The question of the fate of a particular motion is found to be determined by its initial conditions.

A great deal of work has been done on the mathematical theory of the asymptotics of separatrix crossing. [28],[13],[26],[10],[27],[3],[5],[20],[1],[4],[2],[7],[11].

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