Theoretical Study of a Submarine Towed-Array Lifting Device

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Abstract

This work is motivated by submarine use of passive towed sonar arrays of hydrophones. We consider the simplest model of a towed mass. The mass is considered to move only in a horizontal direction x perpendicular to the tow direction. The tension in the tow cable is expected to be nonconstant due to turbulence, and is modeled by a sinusoidal forcing function. The resulting differential equations are analyzed for linear stability and nonlinear dynamical effects.

Introduction

Submarine use of passive towed arrays afford increased sonar capability [1]. The objective here is to deploy a multiline array which can be remotely configured for optimum acoustic sensing capability. That is, a number of individual lines deploy through a single port and fan out to form a three-dimensional, volumetric, array of acoustic sensors. By maintaining a fixed ship bearing and line configuration, composite sensor signals can be analyzed to determine the location and bearing of any acoustic emission source. Deploying and maintaining the position of individual lines comprising a volumetric array requires knowledge of the instantaneous position of each line relative to a fixed point on the ship or relative to the other lines. This must be done in a complex, unsteady ocean environment which is complicated by the turbulent flows associated with the towing vessel and the lines themselves. Aperture generation is currently accomplished through the use of small lifting devices, referred to as 'lateral force devices' or LFDs.

The dynamics of an LFD are complicated by changes in the tow line tension due to flow-induced vibration caused by coherent turbulent structures. These structures result from the turbulent boundary layer on the tow line upstream of the LFD and vortex shedding from the tow line due to crossflow. Full scale experiments in a towing tank have shown that an LFD can exhibit unstable motions under particular conditions. The objective of this work is to provide analytical and numerical modeling of this effect, and to determine parameter ranges in which instabilities will result in undesirable large amplitude motions of the LFD and accompanying temporal variations in the array aperture.

Simpified Model

We investigate the properties of a simplified model of a towed LFD. The coordinate x is used to measure the horizontal location of the LFD perpendicular to the tow direction, see Figure 1. We consider two cases: a) in which the tow cable is rigid, and hence can withstand compressive loads, and b) in which the tow cable is incapable of compression. In both cases, the tension T in the tow cable is expected to be nonconstant due to turbulence. In case a), T is modeled by a sinusoidal forcing function, $T = T_0 + T_1 \cos \Omega t$, while in case b) we assume $T = (T_0 + T_1 \cos \Omega t) H(T_0 + T_1 \cos \Omega t)$ where $H(\cdot)$ is the Heaviside step function. This leads to the approximate differential equation:

$$m\ddot{x} + T\frac{x}{L} + c\dot{x}|\dot{x}| = c_0 U(t)^2 \tag{1}$$

Here the force exerted by the cable on the LFD is Tx/L. We neglect changes in the length L of the tow cable and treat L as a constant. The LFD is treated as a plate oriented so that a normal to the plate's surface points in the x-direction. The term $c\dot{x}|\dot{x}|$ is a fluid drag force and the term $c_0 U(t)^2$ is a fluid lift force. For mathematical simplicity in what follows, we assume that the lift force is negligible, an assumption which is equivalent to assuming that the angle of attack of the plate with respect to the tow direction is zero.

Eq.(1) may be rescaled to take on one of the forms:
Case a) (rigid cable):

$$\ddot{x} + (\delta + \epsilon \cos t)x + \dot{x}|\dot{x}| = 0 \tag{2}$$

Case b) (flexible cable):

$$\ddot{x} + (\delta + \epsilon \cos t)\ H(\delta + \epsilon \cos t)x + \dot{x}|\dot{x}| = 0 \tag{3}$$

Eq.(2) is a nonlinear Mathieu equation and eq.(3) is a nonlinear Hill's equation. In this paper we present a linear stability analysis of both eqs.(2) and (3), and a nonlinear analysis of eq.(2).

Although eqs.(2) and (3) offer simplified models of the real world situation, they are nevertheless nonlinear differential equations and as such may be expected to support a variety of nonlinear phenomena including instability due to subharmonic resonance and related bifurcations, as well as chaos.

The dynamics of eqs.(2) and (3) are determined by the values of the parameters δ and ϵ. We shall be interested in describing the dynamics of eqs.(2) and (3) in the region of the $\delta - \epsilon$ parameter plane for which $0 < \delta < 1$ and $0 < \epsilon < 3$. This region is chosen because it includes some important dynamical effects, including a well-known instability tongue emerging from the point $\delta = \frac{1}{4}, \epsilon = 0$, which is due to the 2:1 subharmonic resonance in the linear Mathieu equation.

Linear Stability Analysis

Eq.(2) admits the exact solution $x = 0$. Its stability is governed by the linear Mathieu equation,

$$\ddot{x} + (\delta + \epsilon \cos t)x = 0 \tag{4}$$

A point (δ, ϵ) is said to be stable if all solutions to the linear equation (4) are bounded, and unstable if an unbounded solution exists. The analysis of eq.(4) dates back to the nineteenth century. See [2] for an excellent summary. Regions of instability, called tongues, emerge from the points $\delta = N^2/4$, $\epsilon = 0$, where $N = 0, 1, 2, 3, \cdots$. These tongues correspond to resonances between the natural frequency of the pendulum-like motion of the towed LFD and the forcing frequency in the tow line due to turbulence in the flow. For small values of ϵ, which is related to the amplitude of the forcing function, the largest instability occurs for $N = 1$. See Figure 2 in which the $N = 1$

tongue is displayed, as well as part of the $N = 2$ tongue. Although the linear theory predicts that inside these tongues the motion will become unbounded, the nonlinear term in eq.(2) causes the resonance to detune as the amplitude of x grows. The net effect is the creation of a periodic motion having finite amplitude inside the various tongues, at least for small ϵ.

A similar situation exists for eq.(3). See Figure 2 which also shows the stability chart for eq.(3). Note that eqs.(2) and (3) are identical for $|\delta| \leq |\epsilon|$, in which case the tow cable remains in tension for all t.

Nonlinear Analysis for Small ϵ

For small values of the parameter ϵ, the method of averaging (see e.g. [3]) may be used to obtain approximate expressions for periodic solutions to the nonlinear eq.(2). In the case of the $N = 1$ tongue, we introduce a detuning parameter δ_1:

$$\delta = \frac{1}{4} + \epsilon\delta_1 \tag{5}$$

and we assume a solution of the form:

$$x(t) = R\,\cos\left(\frac{t}{2} + \psi\right) \tag{6}$$

where R and ψ are slowly varying functions of time t. Application of the method of averaging gives the following values for R and ψ which correspond to stable periodic motions:

$$R = \frac{3\pi}{4}\sqrt{1 - 4\delta_1^2}, \qquad \psi = \frac{1}{2}\cos^{-1}(-2\delta_1)\,. \tag{7}$$

This result, which agrees with numerical integration of eq.(2), states that there is an attractive period-2 subharmonic motion at all points inside the $N = 1$ tongue (at least for small ϵ.)

The method of averaging can similarly be applied to $N = 2$ tongue. In this case it is necessary to include terms of order ϵ^2. We omit details but note that the results state that there are two attractive period-1 motions at all points inside the $N = 2$ tongue.

A Secondary Bifurcation

Numerical simulation of eq.(2) shows that as one traverses the right hand boundary C of the $N = 1$ tongue, the change in stability of the $x = 0$ solution (which defines this boundary C) is accompanied by a bifurcation involving the creation of a periodic motion. We define point P on the boundary C to be the point at which the curve C has a vertical tangency. See Fig.3. We find that for points on C below P, a stable period-2 motion is born as we cross C from right to left. However, for points on C above P, an unstable period-2 motion is born as we cross C from left to right. The region of the parameter plane which lies just to the right of C and above P thus contains three steady states: 1) the rest solution $x = 0$, 2) the stable periodic motion which is born as curve C is crossed from right to left below P (and which continues to exist as C is crossed from left to right above P), and 3) the unstable periodic motion which is born as C is crossed from left to right above P. Simulation shows that the stable manifolds of the unstable periodic motion separate those initial conditions which are attracted to the rest solution $x = 0$ from those that are attracted to the stable periodic motion.

In addition to these observations, numerical simulation also shows that there is another bifurcation curve B emanating from point P on which the stable and unstable periodic motions of the preceding paragraph coalesce. On curve B the system exhibits a secondary bifurcation which can be described as a saddle-node of limit cycles. Current work involves application of a perturbation method to obtain an analytic expression for curve B.

Chaos

Numerical simulation of eq.(2) has also shown that there is a region of the parameter plane which involves a variety of complicated bifurcations including a period-doubling route to chaos. This region is shown in Fig.3. Period-doubling occurs along Curve D, and period-quadrupling occurs along curve Q.

Conclusions

We have seen that the simplified model of an LFD displays a wide range of dynamical behavior, depending on the values of the associated parameters. For application to submarines, it is desirable to eliminate as much of the x oscillation of the LFD as possible. To this end, an attractive strategy would be to operate the LFD in a parameter range for which the rest solution $x = 0$ is linearly stable. Our analysis has shown, however, that, at least in the case of a rigid tow strut, this strategy will not necessarily work due to nonlinear effects. In particular, in the case of eq.(2), if the parameters are chosen so as to lie above the curve of secondary bifurcation, see Fig.3, then although the rest solution is stable, there also exists a stable periodic motion. With two stable states present, the one which actually occurs depends on the initial conditions. Since in the application to submarines the initial conditions occur at random, it is impossible to select them so that the desirable $x = 0$ steady state is achieved, even though this state is attractive. It is therefore advisable to avoid such a region of bistability.

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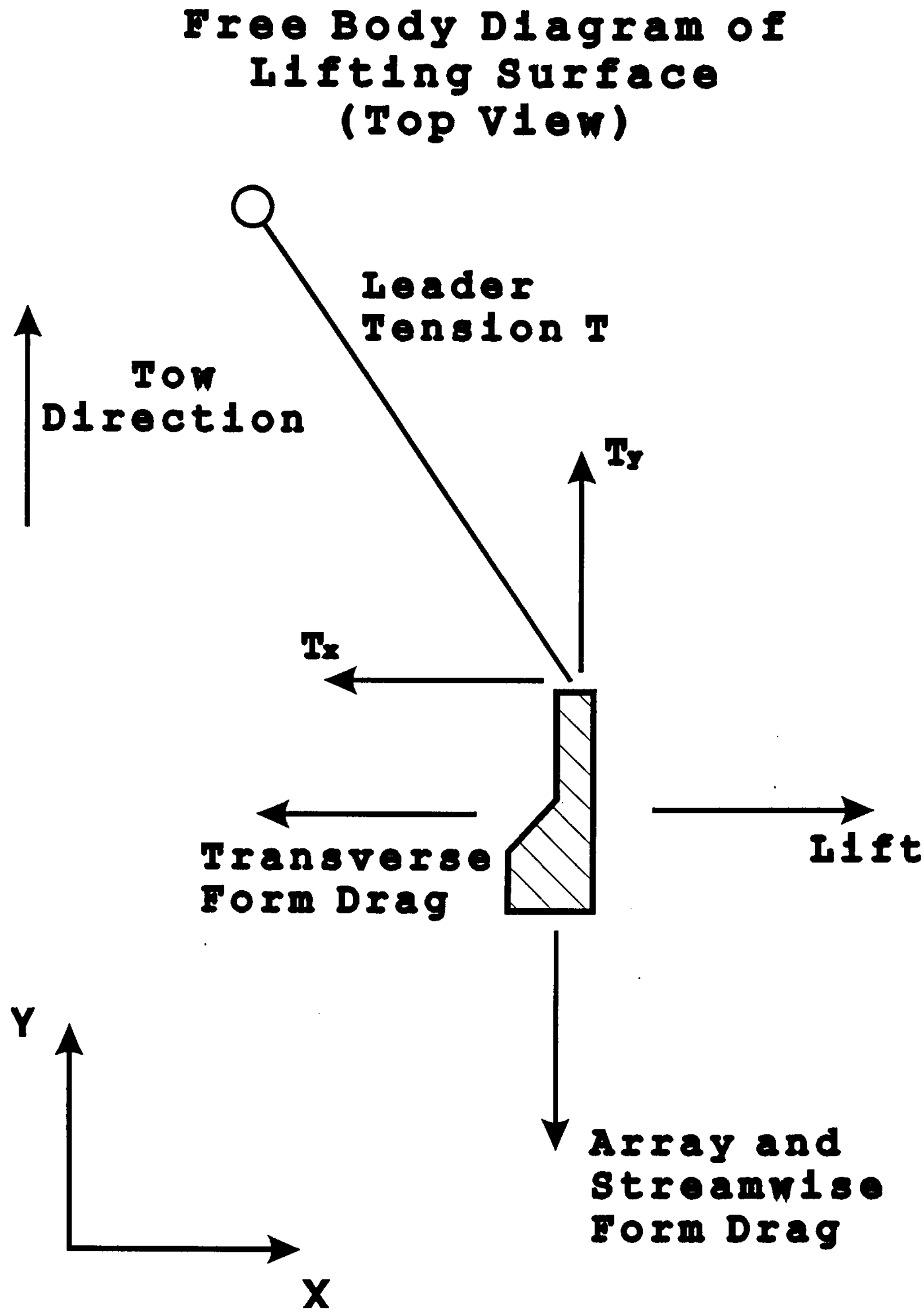

Figure 1. Simplified model of an LFD (lateral force device). The tension in the tow cable is expected to be nonconstant due to turbulence and is modeled by a sinusoidal forcing function. We neglect motion in the y-direction and we treat the cable length L as a constant. The LFD is treated as a plate oriented so that a normal to the plate's surface points in the x-direction. For mathematical simplicity we assume that the lift force is negligible, an assumption which is equivalent to assuming that the angle of attack of the plate with respect to the tow direction is zero.

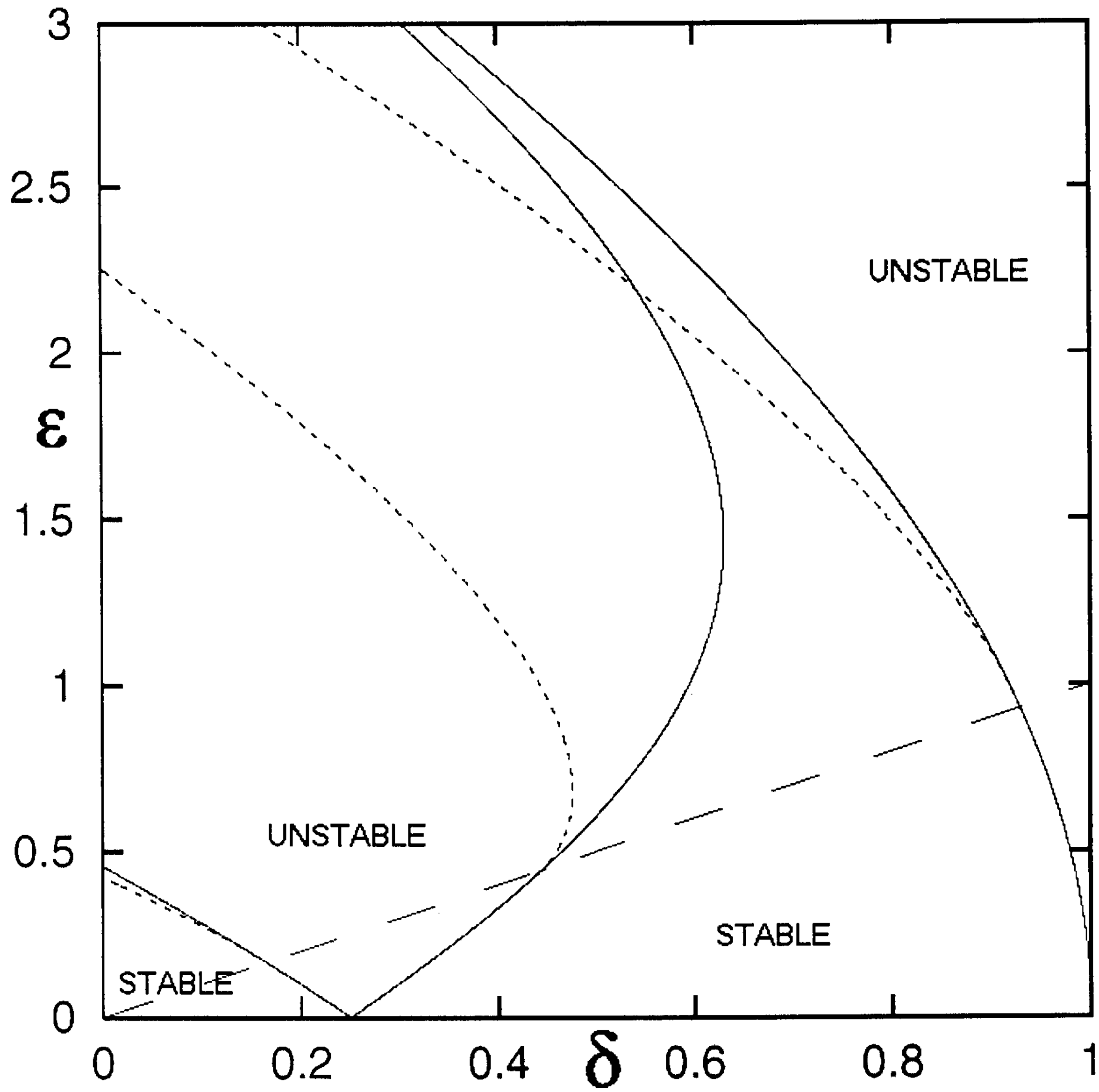

Figure 2. Regions of linear stability and instability in the $\delta - \epsilon$ plane for eq.(2) (solid lines) and eq.(3) (dotted lines). Eq.(3) corresponds to a flexible cable while eq.(2) corresponds to a rigid strut. These equations are identical for points below the straight line $\delta = \epsilon$, which is shown dashed. Points below the dashed line correspond to cases in which the cable remains in tension for all time.

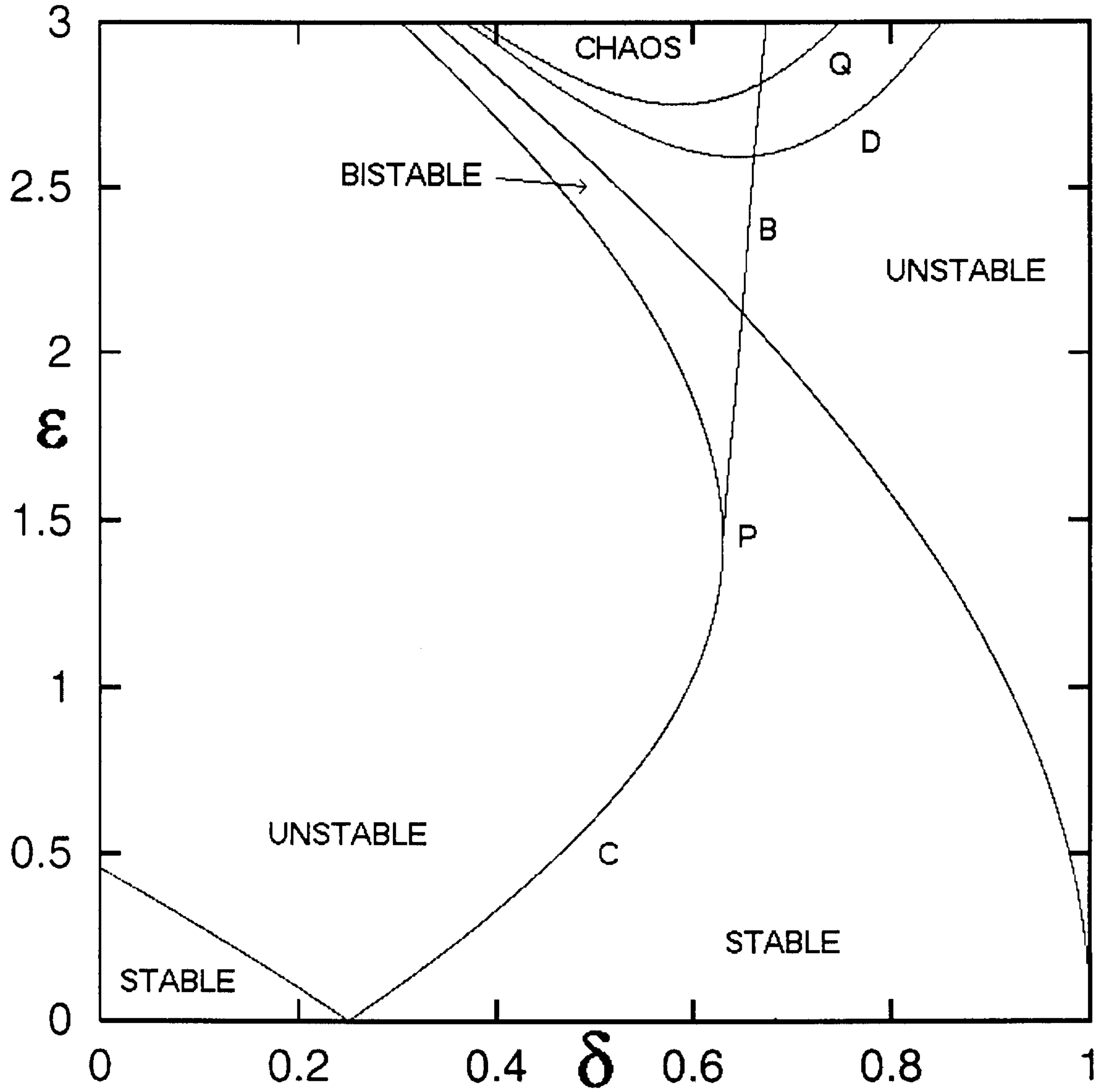

Figure 3. Nonlinear dynamics of eq.(2). Stability of the rest solution $x = 0$ is denoted by "STABLE". Regions marked "UNSTABLE" correspond to unstable rest solution but stable periodic motion. Region marked "BISTABLE" contains both stable rest solution and stable periodic motion. Period-doubling occurs along curve D, and period-quadrupling occurs along curve Q. On curve B the system exhibits a secondary bifurcation which can be described as a saddle-node of limit cycles. See text.