Frequency entrainment for micromechanical oscillator

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We demonstrate synchronization of laser-induced self-sustained vibrations of radio-frequency micromechanical resonators by applying a small pilot signal either as an inertial drive at the natural frequency of the resonator or by modulating the stiffness of the oscillator at double the natural frequency. By sweeping the pilot signal frequency, we demonstrate that the entrainment zone is hysteretic and can be as wide as 4% of the natural frequency of the resonator, 400 times the 1/Q ~ 10^{-4} half-width of the resonant peak. Possible applications are discussed based on the wide range of frequency tuning and the power gain provided by the large amplitude of self-oscillations (controlled by a small pilot signal). © 2003 American Institute of Physics.

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Frequency entrainment is one of the most spectacular phenomena in nonlinear vibrations.1,2 It was discovered in the 17th century by Huygens who remarked that two slightly out of step pendulum-like clocks became synchronized after they were attached to the same thin wooden board. Later, a similar phenomenon (injection locking) was observed in radio-frequency (rf) circuits3–5 and laser systems.6,7

In this letter, we report frequency entrainment observed in a microelectromechanical system (MEMS) in the rf range. Self-sustained vibrations of mushroom-like microfabricated resonators (thin, single-crystal, silicon disk suspended above the wafer by a silicon dioxide pillar at the center) induced by a continuous-wave (cw) laser were reported earlier.8 Large amplitude (~λ/2 laser light) self-sustained mechanical oscillations spontaneously occur at the fundamental frequency f_0 when the intensity of a cw laser beam focused on the perimeter of the disk reaches a critical value P_{threshold}. The self-oscillatory behavior is attributed to the automodulation of the laser-induced thermal stress field that occurs as the disk moves through the interferometric pattern created by incident and reflected laser beams. These spontaneous vibrations are subject to a frequency instability Δff_0~10^{-3} observed on a spectrum analyzer as the sporadic motion of a narrow resonant peak.

We can establish full control over the frequency and phase of the self-oscillations of the disk by applying a small periodic perturbation—a “pilot” signal—via inertial drive (provided by an ac voltage applied to a piezoelement) at a frequency close to f_0 or by modulating the effective spring constant of the disk by superimposing a small rf component on the laser beam intensity8 near 2f_0. As the frequency of the perturbation is tuned close to f_0 or 2f_0, we readily observe a distinct point when the phase of the self-oscillatory motion gets locked to that of the pilot. On the polar plot display of a network analyzer (referenced by pilot signal), the locking is manifested by a change from a pattern of spirals to a single point. Once synchronized, a wide range of frequency entrainment can be demonstrated by detuning the pilot signal from f_0 (or 2f_0) while preserving the locked state. There are no frequency beats in the synchronized state: Only vibrations with the pilot frequency exist in the system.

In our experiment, the entrainment range Δff_0~4% was demonstrated for a MHz-range oscillator.

The fabrication process for rf disklike resonators (disk diameter 30 μm, thickness h=0.25 μm, central SiO_2 pillar 1 μm height, 0.6 μm diameter) has been described elsewhere.9 The resulting structure (inset in Fig. 1) was attached to a piezoelectric actuator and placed in a vacuum chamber (P~10^{-7} Torr). The motion of the disk was detected interferometrically.10

Figure 1 shows the amplitude and frequency of the vibration of the disk as a function of the pilot signal frequency.

![Figure 1](image)

**FIG. 1.** Frequency and amplitude of the laser-induced self-sustained vibrations as a function of the pilot signal frequency. Laser power P_{cw} = 650 μW, perturbation applied as an inertial drive (V_{piezo} = 4 Vol).
A helium–neon (HeNe) laser beam at $P_{\text{laser}} = 650 \mu W$ was used to excite the disk into self-oscillation ($P_{\text{threshold}} \sim 600 \mu W$) and an inertial force perturbation was applied by the piezoeactuator at frequency $f_{\text{pilot}}$. The solid line represents data acquired while linearly increasing $f_{\text{pilot}}$. After establishing synchronization at $f_{\text{pilot}} = f_{\text{lock\_up}}$ ($\sim 2$ kHz below $f_0$), the frequency of the motion follows the change of the pilot frequency. The amplitude of the mechanical motion increases until another critical point is reached ($f_{\text{free\_up}}$, 28 kHz above $f_0$). Synchronization is then broken, self-oscillations at the natural frequency $f_0$ are restored and for $f_{\text{pilot}} > f_{\text{free\_up}}$ the pilot signal is ineffective. This general sequence is repeated on a down sweep, except that when entrained at $f_{\text{pilot}} < f_{\text{lock\_down}}$, the oscillation amplitude is reduced. The critical values of the pilot frequency $f_{\text{free}}$ and $f_{\text{lock}}$ are different for up and down sweeps, demonstrating the hysteretic behavior of the synchronization for our micromechanical oscillator.

The width of the entrainment range ($f_{\text{lock}} - f_{\text{free}}$) is determined by the power of the pilot signal. Figure 2 shows an entrainment map where critical points (for up and down sweeps) are plotted as a function of the ac piezovoltage. This plot has a V-shape, consistent with the expectation of zero entrainment range at zero perturbation. Due to the hysteresis, two entrainment areas are present, corresponding to up and down frequency sweeps. These two areas overlap above the natural frequency, creating an “inner V”—where synchronization to the pilot signal is assured. After establishing phase lock, it is preserved for pilot excursions within the “outer V”—boundaries marked by $f_{\text{free\_down}}$ and $f_{\text{free\_up}}$, as demonstrated by applying frequency and phase modulation to the pilot signal.

Subharmonic or superharmonic entrainment expected at $nf_0$ and $(1/n)f_0$ ($n$ is an integer) could be more attractive for applications because the pilot signal and the base frequency are well separated, avoiding parasitic coupling. We failed to demonstrate entrainment with these methods using the inertial drive as a pilot signal. However, a wide range of subharmonic entrainment was observed by partial modulation of the laser beam intensity at $f_{\text{laser}} = 2f_0$.

Figure 3 shows the map for $2f_0$ entrainment. A cw HeNe laser, $P_{\text{laser}} = 1.7 \text{ mW}$ was focused on the periphery of the disk to initiate self-oscillations. An Ar+ ion laser beam modulated at $f_{\text{pilot}} = 2f_0$ illuminated the area around the pillar providing parametric excitation. The ac component of the Ar+ laser intensity is plotted as the vertical axis. Hysteretic behavior of the parametric synchronization, similar to that for the inertial-drive entrainment is illustrated by the inner V, i.e., overlap of the entrainment regions from up and down pilot frequency sweeps. The outer V, the range of frequency detuning that still preserves synchronization, is expanded to 70 kHz for 50 $\mu W$ pilot signal power. Frequency stability of the entrained oscillations seems to be limited by that of the pilot generator. In our case, $\Delta f/f_0 \sim 10^{-9}$ was observed.

The analysis of frequency entrainment in our mushroom-like oscillators can be obtained from the differential equations describing self-oscillatory motion:12

\[ \ddot{z} + \frac{1}{Q} (\dot{z} - DT) + (1 + CT) (z - DT + \beta (z - DT)^3) = M \sin(\omega_{\text{piezo}} t), \]  
(1)

\[ T = A P_{\text{laser}} [1 + \dot{\varphi} \sin(\omega_{\text{mod}} t)] \cdot [\alpha + \gamma \sin^2 2\pi (z - z_0)] - BT. \]  
(2)

The disklike resonator is represented in Eq. (1) as a one degree of freedom oscillator with quality factor $Q$, temperature-dependent stiffness $(1 + CT)$, and nonlinear coefficient $\beta$. Time is nondimensionalized by the small amplitude period of vibration. $T$ is the temperature increase under the laser spot and $z$ is the deflection normalized by the laser wavelength $\lambda$. The $DT$ terms represent deflection of the disk due to heating. They arise from the slightly bowed ($\sim 50 \text{ nm}$) equilibrium shape of the disk (a result of the residual stress incorporated in the SOI wafer). The term on the right-hand side of Eq. (1) corresponds to the inertial pilot signal applied by the piezoelement at $\omega_{\text{piezo}}$. Equation (2) describes the heat balance for the mushroom. The incident laser power $P_{\text{laser}}$ can be partially modulated to produce a pilot signal at a frequency $\omega_{\text{mod}}$ ($\varphi$ = modulation depth). The term on the second set of square brackets describes the portion of the laser power that is actually absorbed by the disk:

A constant minimum absorption $\alpha$, and oscillatory compo-
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