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Thermo-Mechanical Coupling Mechanism in NEMS Resonators

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Abstract: An optically thin resonator suspended over a substrate and illuminated with a laser forms an interference field that couples deflection of the resonator to absorption within it. For high enough laser powers, such resonators have been shown to be interferometrically transduced to stable oscillations due to the feedback between heating and deflection. The coupling between thermal stresses, deflection, and absorption is the driving mechanism of such oscillations. In this work we study the deflection of doubly supported micro-beams under steady state heating, and illustrate the role of thermal strains and deformation of the beam supports on deflection and ultimately on the laser power level needed to induce stable oscillations.

1. Introduction: Due to their sharply peaked resonance curves and high operating frequencies, resonant microelectromechanical systems (MEMS) have been integrated with traditional electronics in the past decade and used in signal processing[1-2] and sensing applications[3]. MEMS beams are typically fabricated out of thin silicon films and driven using electrostatic actuation[4]. This actuation technique requires an externally modulated drive signal and conductive layers fabricated on the device which add unwanted damping. Interferometric transduction is a technique which skirts these drawbacks and can be used to inducing oscillations[5-7].

Property	Si	SiO_2
ρ [kg/m^3]	2420	2200
ν	0.279	0.17
E [GPa]	130	70
α_T [K^{-1}]	2.5×10^{-6}	0.5×10^{-6}
K [W/m*K]	170	1.38
C [J/kg*K]	712	1120

Table 1: Material properties used in FEM analyses

In laboratory experiments, samples are mounted in a vacuum chamber evacuated to $<10^{-6}$ mbarr to reduce viscous damping and an unmodulated (CW) laser is focused on the center of the beam. The reflected signal is measured with a high speed photodiode and analyzed on a spectrum analyzer (see Figure 1). The beam-gap-substrate system forms a Fabry-Pérot interferometer. As center of the beam deflects, the amount of absorption and thus heating changes. For low laser power, the beam will deflect statically, but for $P>P_H$, static deflection becomes unstable and the beam will begin to self-oscillate. Models[5-6,8-11] of these oscillations have been analyzed to determine the threshold power for self-oscillation, but little work has been done to understand the thermal-mechanical mechanism responsible or to systematically study parameters in the models. Interferometric transduction depends on the feedback between heating and displacement, yet a beam theory model for an initially straight beam would suggest that there would be no out of plane displacement due to heating in a doubly supported beam until the buckling load is reached. A simple analysis shows that through thickness temperature gradient in the beam is negligible thus no bending moment would occur. Furthermore, for devices studied here radiation pressure produces deflections on the order of 10^{-4}Å, insufficient to drive the motion. To investigate the cause of this deflection, we build a finite element method (FEM) model of a doubly-supported beam subject to steady state heating at its center. We show that deflection due to heating comes from compressive stresses at the beam's support, and

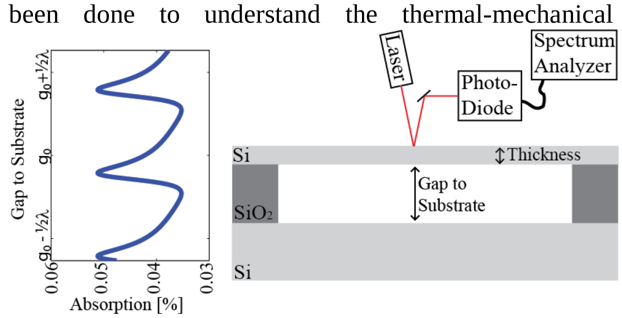


Figure 1: Side view of a MEMS resonator illuminated with a laser to form a Fabry-Pérot interferometer which couples absorption to center deflection.

illustrate the importance of this deflection in driving opto-thermal limit cycle oscillations.

2. Thermal/Structural FEM Modeling: Our analysis models doubly clamped beams of length [7,10,15 & 20] μm, fabricated out of silicon-silicon dioxide-silicon SOI wafers, and measured to be 201 nm thick and 2 μm wide with 2 μm of undercutting. First vibration mode resonant frequencies were measured to be [17,10, 5 & 3.6] MHz respectively using a set-up schematically depicted in Figure 1. FEM analysis of the structures and resonant frequencies indicates the presence of ~15MPa compressive pre-stress in the devices, an unavoidable byproduct of the SOI wafer fabrication process.

An FEM model of each beam is built incorporating a large portion of the surrounding substrate. Quarter symmetry is used to reduce the computational burden. Since devices operate in vacuum

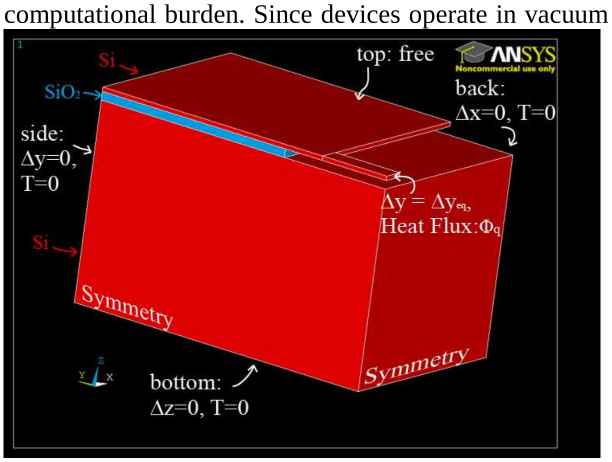


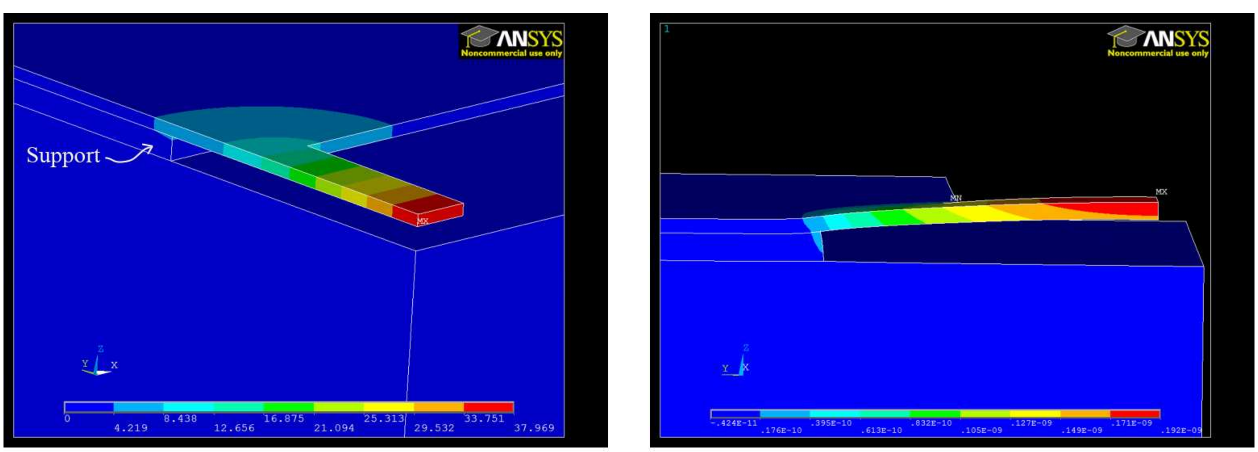
Figure 2: Material assignments as well as thermal and structural boundary conditions.

and temperature changes are small, convective and radiative heat loss is negligible. The laser power is assumed to be absorbed evenly throughout the thickness of the device, and the substrate outside of the model region is assumed to be an infinite heat sink. Thus, the temperature is specified (T=0) on the bottom and sides, zero flux is specified along the top and symmetry surfaces and a 1mW equivalent flux, Φ_q, is applied to the beam center cross-section (see Figure 2). Note that 1 mW is representative of the power levels used in experiments. To incorporate the effects of pre-stress, an equivalent step displacement, Δy_{eq}, is applied to the center cross-section. The top surface is free, and the bottom, side and back constrained to prevent translations and rotations of the model. A mesh convergence study ensures displacements are within 2% of the true results. Material properties used are listed in Table 1.

The analysis proceeds in two steps. First, the temperature field T(x,y,z) is computed. Then the temperature is input into the structural analysis which computes the resulting thermal strains and deformation.

3. Results: Interferometric transduction depends on the feedback between heating and displacement. However, an Euler-Bernoulli beam theory model of heating in an initially straight doubly supported beam would suggest that there is no out of plane displacement in the beam before thermal buckling, regardless of the temperature - thus that heating and displacement are uncoupled. If realized in a model of interferometric transduction, this idea would suggest that self-oscillation is not possible until the laser power is high enough to thermally buckle the beam at which point greater compressive stress due to heating would arch the structure, coupling displacement to heating.

FEM analysis shows deflection due to heating before thermal buckling as a result of two competing forces. As has already been noted in a similar study of cantilevered beams[12-13], the thermal conductivity of silicon is much higher than that of the underlying silicon oxide. As a result, temperature changes are largely confined to the device layer causing large vertical thermal gradients at the interface of the device layer and oxide layer near the support. These thermal gradients lead to differing horizontal strains in the device layer and oxide layer. The mismatch in thermal expansion



(a) Temperature

(b) Displacement (UZ)

Figure 3: Results of steady state thermal and small deflection structural analyses for 7μm beam. Temperature in degrees Kelvin above ambient is given in (a) and vertical displacement in meters is given in (b).

coefficients between the two layers (see Table 1) adds to this effect to create a bending moment that tends to rotate the beam downwards towards the substrate. However, in doubly supported beams compressive stresses across the length of the device due to heating has the opposite effect. Note that pre-stress in the device causes compressive stresses at the support. Since the device layer is joined to the underlying oxide layer, shear stresses develop along the bottom side of the device layer in the region of the support. These stresses, offset from the beam centerline, tend to arch the beam up away from the substrate. Heating of the beam causes further compressive stress due to thermal expansion which amplifies this affect. For the beams studied, the thermal compressive force dominates, causing the beams to arch away from the substrate when heated. Given our assumptions of small displacement linear thermoelasticity, the displacement due to heating grows linearly with temperature. We define the thermo-mechanical coupling of the beam as

$$D = \frac{UZ_{center}}{T_{ave}}$$

where UZ_{center} is the vertical component of the displacement at the center of the beam, and T_{ave} is the average temperature above ambient in the beam. Note that in calculating the coupling, we include only the displacement due to heating, and not the portion due to pre-stress. See Figure 3 for an illustration of the results and Table 2 for a list of calculated thermo-mechanical coupling values.

Length [µm]	7	10	15	20
D [pm/K]	2.58	3.51	5.08	6.48

Table 2: Thermo-mechanical coupling of beams by length

4. Quasi-Static Modeling of Self-Oscillation: Variations of the same quasi-static model for self-oscillation have been presented before[6,8-11] and analyzed in the context of specific devices: beams, disks, domes, etc. Further motivation and details on the derivation are available in the references, and in this work we simply present the model in order to illustrate the effect of direct displacement due to heating on self-oscillation. Although beams have displacements that vary in space, the first vibration mode is assumed and the displacement of the center of the beam is modeled as a one degree of freedom oscillator. The average temperature in the device is modeled using a lumped thermal model. The absorption is assumed to be periodic in half the wavelength of the laser. Model equations are given below, where time is normalized by the resonant frequency of the device, and length is normalized by the laser wavelength:

$$\ddot{z} + \frac{\dot{z}}{Q} + (1 - CT)z + \beta z^3 = DT$$
$$\dot{T} = -BT + HP[\alpha + \gamma \sin^2(2\pi(z - z_o))]$$

The variable z is the normalized center displacement and T is the average temperature in the device. In the mechanical equation, the parameter Q is the mechanical quality factor, C gives the frequency dependence on temperature, β is the cubic stiffness due to membrane stresses, and D is the thermo-mechanical coupling. Note that the DT term in the mechanical model gives direct displacement due to heating. In the thermal equation, B is the cooling rate due to conduction, H is the inverse of the lumped thermal mass and P is the laser power. The parameters α,γ and z_o describe absorption in the device due to the interferometric effect depicted in Figure 1. Parameters are fit using a combination of experimental results, analytic calculations, and FEM modeling.

Equilibrium and periodic solutions of the model are calculated using the numerical continuation tool AUTO2000[14-15]. Numerical continuation allows us to efficiently determine whether self-oscillation is possible in the model for a given laser power. A representative bifurcation diagram is given in Figure 4 using parameters calculated for the 15 µm beam listed in Table 3.

Note the presence of hysteresis loops, a phenomenon which is seen experimentally[6]. As the laser power is increased from zero, we reach a power (P_H) at which the beam spontaneously goes into self-oscillation at a finite amplitude. Once vibrating, the oscillation will continue as the power is reduced until at a lower power (P_L) vibration ceases. We also see that despite the low thermo-mechanical coupling of ~5[pm/K], direct displacement due to heating reduces the power at which we first see self-oscillation (P_H) by a factor of ~3.5, and the lowest power for which self-oscillation is possible (P_L) by a factor of ~30. This effect is more pronounced for shorter beams which have higher buckling loads.

Parameter	Value	Parameter	Value
Q	10,900	ω_r	5.0 [MHz]
H	5,570 [K/W]	B	0.112
α	0.035	γ	0.011
β	6.72	C	0.0159 [K^{-1}]
D	8.04×10^{-6} [K^{-1}]	P	Variable

Table 3: Model parameters for 15µm beam

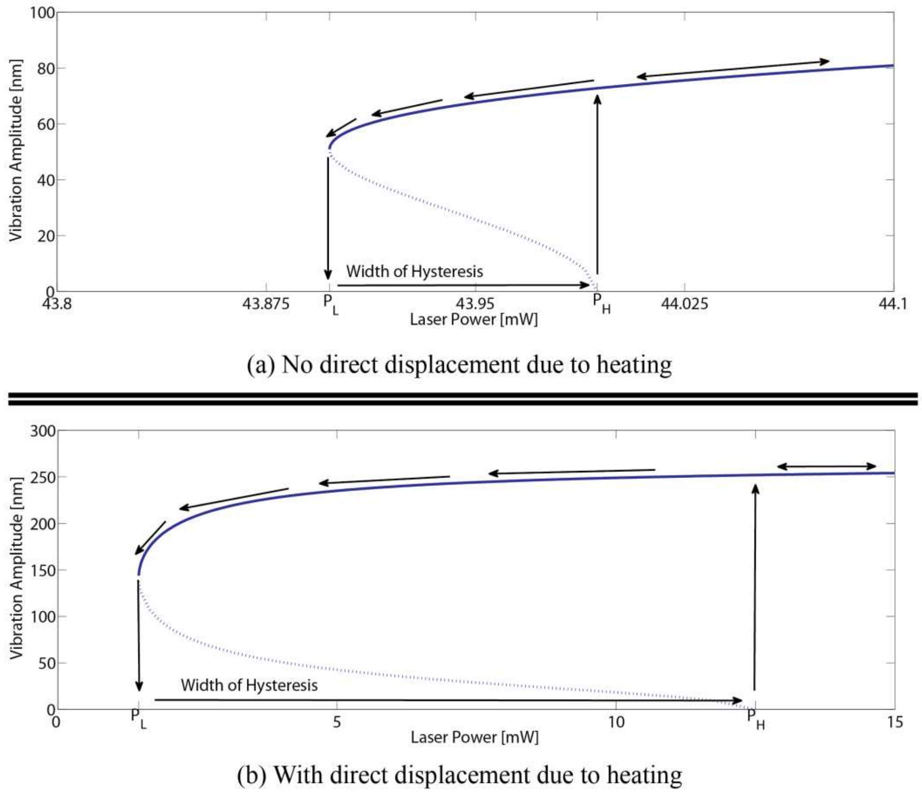


Figure 4: Numerical continuation of periodic solutions. Solid lines represent stable motions, dashed lines represent unstable motions. Without direct displacement due to heating, self-oscillation does not occur until the laser power is P=44mW, slightly greater than the power at which the beam buckles, P=28mW. Direct displacement due to heating reduces the power at which oscillation is first seen (P_H) and the reduces the lowest power for which oscillation is possible (P_L)

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