Abstract for International Conference on Structural Nonlinear Dynamics and Diagnosis, Marrakech 2016.

On the Treatment of Delay Terms in the Slow Flow of Nonlinear Differential Equations with Delayed Self-Feedback

Si Mohamed Sah*[1] and Richard H. Rand[2]

[1]Nanostructure Physics, KTH Royal Institute of Technology, Stockholm, Sweden.

[2]Dept. of Mathematics, Dept. of Mechanical & Aerospace Engineering, Cornell University. Ithaca, NY 14853, USA.

Abstract

This work concerns the dynamics of nonlinear systems that are subjected to delayed self-feedback. Perturbation methods applied to such systems give rise to slow flows which characteristically contain delayed variables. We consider two approaches to analyzing Hopf bifurcations in such slow flows. In one approach, which we refer to as approach **I**, we follow many researchers in replacing the delayed variables in the slow flow with non-delayed variables, thereby reducing the DDE slow flow to an ODE. In a second approach, which we refer to as approach **II**, we keep the delayed variables in the slow flow. By comparing these two approaches we are able to assess the accuracy of making the simplifying assumption which replaces the DDE slow flow by an ODE. We apply this comparison to two examples, Duffing and van der Pol equations with delayed self-feedback.

For example take the Duffing equation with delayed self-feedback.

$$\ddot{x} + x = \epsilon \left[-\alpha \dot{x} - \gamma x^3 + k \, x_d \right] \tag{1}$$

where $x_d = x(t-T)$, where $T =$ delay. We treat eq.(1) with the two variable perturbation method, where $x(\xi, \eta)$, where $\xi = t$ and $\eta = \epsilon t$. We expand $x = x_0 + \epsilon x_1 + O(\epsilon^2)$ and obtain the following equation on x_0:

$$L x_0 \equiv x_{0\xi\xi} + x_0 = 0 \quad \Rightarrow \quad x_0(\xi, \eta) = A(\eta)\cos\xi + B(\eta)\sin\xi \tag{2}$$

*smsah@kth.se

Eliminating secular terms in the x_1 equation gives the following slow flow:

$$\frac{dA}{d\eta} = -\alpha\ \frac{A}{2} + \frac{3\ \gamma\ B^3}{8} + \frac{\gamma\ A^2 B}{8} - \frac{k}{2} A_d \sin T - \frac{k}{2} B_d \cos T \tag{3}$$

$$\frac{dB}{d\eta} = -\alpha\ \frac{B}{2} - \frac{3\ \gamma\ A^3}{8} - \frac{\gamma\ AB^2}{8} - \frac{k}{2} B_d \sin T + \frac{k}{2} A_d \cos T \tag{4}$$

where $A_d = A(\eta - \epsilon T)$ and $B_d = B(\eta - \epsilon T)$ are delay terms in the slow flow.

Method I involves replacing the delay terms A_d, B_d in the slow flow (3),(4) respectively by undelayed terms A, B, resulting in the slow flow ODEs:

$$\frac{dA}{d\eta} = -\alpha\ \frac{A}{2} + \frac{3\ \gamma\ B^3}{8} + \frac{\gamma\ A^2 B}{8} - \frac{k}{2} A \sin T - \frac{k}{2} B \cos T \tag{5}$$

$$\frac{dB}{d\eta} = -\alpha\ \frac{B}{2} - \frac{3\ \gamma\ A^3}{8} - \frac{\gamma\ AB^2}{8} - \frac{k}{2} B \sin T + \frac{k}{2} A \cos T \tag{6}$$

Method II involves studying the slow flow (3),(4) as it is.

Figure 1 shows a comparison, in the case of Duffing equation, between the analytical Hopf conditions obtained via the two approaches and the numerical Hopf curves. The approach **II** plotted by red/dashed curves gives a better result than the approach **I** (black/dashdot curves). Therefore in the case of Duffing equation, treating the slow flow as a DDE gives better results than approximating the DDE slow flow by an ODE.

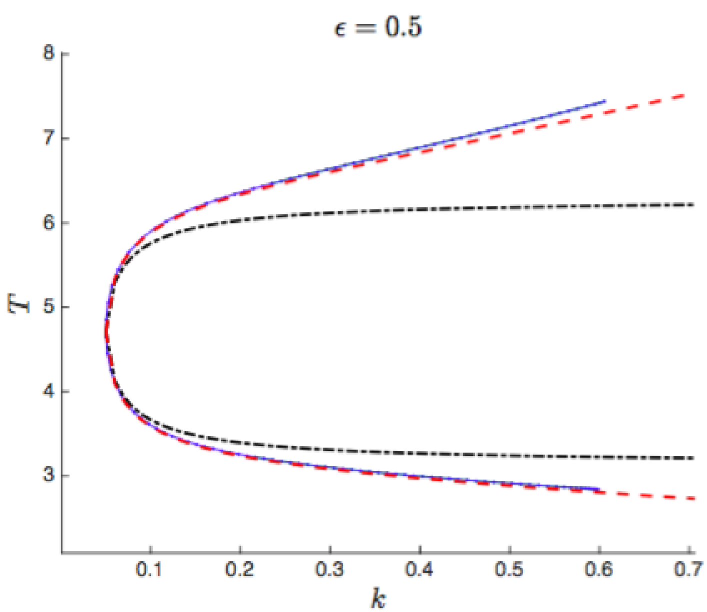

Figure 1: Numerical Hopf bifurcation curves (blue/solid) for Duffing equation. Also shown are the results of approach **I** (black/dashdot), and the results of approach **II** (red/dashed)