1. Let \((x_n)_{n=1}^N\) and \((y_n)_{n=1}^N\) be two finite length sequences of positive real numbers. Denote by \((x^*_n)_{n=1}^N\) and \((y^*_n)_{n=1}^N\) the same sequences, but written in a decreasing order. Show that 
\[
\sum_{n=1}^N x_n y_n \leq \sum_{n=1}^N x^*_n y^*_n.
\]

2. Let \(a\) and \(b\) be positive integers whose greatest common divisor \(\gcd(a, b)\) is 1. Show that the equation
\[
am + bn = ab - a - b
\]
ever has positive integer solutions \(m\) and \(n\).

3. Let \(1 > \epsilon > 0\) be fixed. Show that every real number can be approximated arbitrarily well by differences of the form
\[
n^{1-\epsilon} - m^{1-\epsilon}
\]
for some positive integers \(m\) and \(n\).

By *arbitrarily good approximation of \(x\), we mean that given any \(\delta > 0\), there are positive integers \(m\) and \(n\) so that
\[
|x - (n^{1-\epsilon} - m^{1-\epsilon})| < \delta.
\]

4. Let \(k\) be the smallest positive integer with the following property:

There are distinct integers \(m_1, m_2, m_3, m_4,\) and \(m_5\) such that the polynomial
\[
p(x) = (x - m_1)(x - m_2)(x - m_3)(x - m_4)(x - m_5)
\]
has exactly \(k\) nonzero coefficients.

Find, with proof, a set of integers \(m_1, m_2, m_3, m_4,\) and \(m_5\) for which this minimum is achieved.

5. For any two positive numbers \(a\) and \(b\), find the side length of the smallest square containing two non-overlapping circles of respective radii \(a\) and \(b\). (By non-overlapping, we mean that the interiors of the two circles do not intersect.)