1. 2004 exam; easy probability.
2. easy analytic geometry.
3. obvious fixed point, can use monotonicity to show existence.
4. count powers of 2 and 5.
5. easy product of geometric series formula based on prime factorization.
6. 1999 exam presumably routine.
7. 2004 exam; also Engel Chapter 1 E8; solvable by arguing it is always possible to reduce the number of hostile couples using pigeonhole and idea of reversing positions along a suitable arc.
8. 2004 exam; also Engel Chapter 2 #13; pigeonhole applied to a big enough rectangle of lattice points; two of the rows must have the same color, and one can continue form there.
9. 2005 exam; modular arithmetic with some size estimates.
10. 2005 exam; easy intermediate value theorem argument for existence of a fixed point, then one possibility is an MVT based contraction argument to show convergence.
11. 2005 exam; consider incremental reaarangements to show max must involve only products of 2’s and 3’s; then find the best such product.
12. 2005 exam; working from the bottom counting paths is straightforward.
13. 2004 exam; equality when $a = b$ and can easily estimate the sign of (say) the derivative with respect to $a$ of $\ln(\text{RHS}) - \ln(\text{LHS})$.

14. 2005 exam; integration by parts is the easiest way to show this.

15. 2004 exam; Little Fermat and a bit of factoring.

16. 2004 exam; in vector terms $A_{i-1} + A_i = B_{i-1} + B_i$, and it’s easy to solve for $B_n$. 