Problem 1. A box contains 7 white balls and 8 black balls. If 3 balls are drawn from the box at random, what is the probability of drawing 2 white balls and 1 black ball?

Problem 2. The lines normal to the parabola \( y = x^2 \) through the points \((x, x^2), x \neq 0\), intersect the y-axis at a set of points \(Y\). What is the largest value of \(y\) that is NOT in the set \(Y\).

Problem 3. Put \(c_1 = 1\) and define \(c_{n+1} = c_n + \frac{1}{c_n}\). Find \(\lim_{n \to \infty} c_n\).

Problem 4. How many 0’s does 2000! end in?

Problem 5. Find the sum of all positive integral divisors of 9600.

Problem 6. A rectangle with sides parallel to the axes is drawn in the first quadrant region bounded by the \(x\)-axis and the curve \(y = \frac{x}{x^2+1}\). The rectangle is then rotated about the \(x\)-axis to form a solid. What is the maximum possible volume of this solid?

Problem 7. The UN invited \(2n\) diplomats for a dinner. Each of them has at most \(n - 1\) enemies. Show that you can seat them at a round table so that nobody sits next to an enemy. (We assume that being an enemy is symmetric: if \(A\) is an enemy of \(B\), then \(B\) is an enemy of \(A\).)

Problem 8. Every point in the plane is colored either blue, red, or green. Show that there is a rectangle all of whose corners have the same color.

Problem 9. The number 313726685568359708377 is the 11\(^{th}\) power of some number \(n\). Find \(n\).
Problem 10. Starting with any real number $x_0$, a sequence of numbers $\{x_n\}_{n=0}^{\infty}$ is defined by $x_{n+1} = \cos(x_n)$ where $x_n$ is measured in radians. Show that

$$\lim_{n \to \infty} x_n$$

exists and is independent of the initial value $x_0$.

Problem 11. The sum of the positive integers $\{a_1, \ldots, a_r\}$ is $a_1 + \ldots + a_r = 2005$. What is the largest possible product $a_1 \cdot \ldots \cdot a_r$ that can be formed under this condition?

Problem 12. A path from $A$ to $B$ in the figure is valid if it does not cross itself, and never moves downwards. How many valid paths are there?
Problem 13. Prove the inequality
\[
\left( \frac{a}{b} \right)^b \leq \left( \frac{a + 1}{b + 1} \right)^{b+1}
\]
for \( a > 0, \ b > 0 \).

Problem 14. Show that

\[
\lim_{R \to \infty} \int_0^R \sin (t^2) \, dt
\]
exists and is finite.

Problem 15. Let \( p \) be a prime number.

(a) Show that \( p \) divides \( 2^p - 2 \).

(b) Show that \( p^2 \) divides \( 2^{p^2} - 2^p \).

Problem 16. Given are \( n \) points in space: \( A_0, A_1, \ldots A_n = A_0 \). Pick a point \( B_0 \) and then pick points \( B_1, B_2, \ldots B_n \) such that for each \( k = 1, 2, \ldots, n \) the line segments \( A_{k-1}A_k \) and \( B_{k-1}B_k \) have common mid points.

Show that \( B_0 = B_n \) provided that the number of points \( n \) is even.