1. (30 pts) Investigate the behavior of the function: find the critical points, the intervals of increase and decrease and the intervals where the graph is concave up and down. Determine the local maxima and minima and the points of inflection. Sketch the graph.

\[ y(x) = (x^2 - x + 1)e^x \]

\[ y'(x) = (2x - 1)e^x + (x^2 - x + 1)e^x = (x^2 + x)e^x \]

\[ y''(x) = (2x + 1)e^x + (x^2 + x)e^x = (x^2 + 3x + 1)e^x \]

Critical points:
\[ y'(x) = 0 \]
\[ e^x > 0 \text{ for all } x \]
\[ (x^2 + x)e^x = 0 \]
\[ x^2 + x = 0 \]
\[ x(x+1) = 0 \]

CPs:
\[ x = 0, \quad x = -1 \]

Increasing/decreasing:
\[ CP \ -1: \ \text{inc} \rightarrow \text{dec}, \quad \text{local max} \]
\[ CP \ 0: \ \text{dec} \rightarrow \text{inc}, \quad \text{local min} \]

Or 2nd derivative test:
\[ y''(x) = (x^2 + 3x + 1)e^x \]
\[ y''(-1) = -e^{-1} < 0 \quad \text{local max} \]
\[ at \ -1 \]
\[ y''(0) = e^0 = 1 > 0 \quad \text{local min} \]
\[ at \ 0 \]

Concavity:
\[ y'' = 0, \quad (x^2 + 3x + 1)e^x > 0 \]
cannot be factored

\[ x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1}}{2} = \frac{-3 \pm \sqrt{5}}{2} \]
\[ \frac{-3 - \sqrt{5}}{2}, \quad \frac{-3 + \sqrt{5}}{2} \]

Concavity changes
\[ at \ -3 - \frac{\sqrt{5}}{2} \quad \text{and} \quad -3 + \frac{\sqrt{5}}{2} \]
so both are inflection points