Precise large deviation probabilities for random walks with stationary heavy tailed steps

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Abstract

This is joint work with Olivier Wintenberger (Paris Dauphine). We study precise large deviation probabilities in the spirit of A.V. and S.V. Nagaev; see A.V. Nagaev (1969) Theory Probab. Appl. 14, 51–64 and 193–208, and S.V. Nagaev (1979) Ann. Probab. 7, 745–789. They studied random walks of iid steps with a regularly varying right tail and showed that the right tail of the random walk at a given time is equivalent to the tail of the maximum step up to this time. In this talk, analogs are provided for random walks generated from a strictly stationary step sequence. The dependence structure is rather general, but excludes long range dependence. In particular, analogs of Nagaev’s theorem can be derived for Markov chains and return models for speculative prices (GARCH, stochastic volatility model). The general framework for these results is regular variation of the finite-dimensional distributions of the step sequence. The proofs use ideas of Adam Jakubowski developed for the central limit theory with infinite variance stable limits. Precise large deviations can be used, for example, to derive precise bounds for ruin probabilities for such random walks. For linear regularly varying processes this approach was chosen in T. Mikosch and G. Samorodnitsky (2000) Ann. Appl. Probab. 10, 1025–1064, and for solutions to affine stochastic difference equations in D. Buraczewski, E. Damek, T. Mikosch and J. Zienkiewic (2011) (2011). The results of this talk generalize the mentioned papers and also give insight how large deviations occur in a random walk with dependent heavy tail steps.