On the existence of paths between points in high level excursion sets of Gaussian random fields

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Abstract

The structure of Gaussian random fields over high levels is a well researched and well understood area, particularly if the field is smooth. However, the question as to whether or not two or more points which lie in an excursion set belong to the same connected component has eluded analysis. We study this problem from the point of view of large deviations, analyzing the asymptotic probabilities that two such points are connected by a path laying within the excursion set, and so belong to the same component. This problem turns out to be intimately related to the problem of finding minimal energy measures with respect to the covariance kernel of the field. We characterize such measures, and prove that the optimal (most likely) paths are, in fact, the minimal capacity paths. We will conclude with considering the case of two points far away from each other, and observing the difference between the short and long memory cases.

This is joint work with R. Adler and E. Moldavskaya.