1a. By the Fundamental Theorem of Calculus we have
\[
\frac{d}{dx} \int_0^x \cos t \, dt = \cos x.
\]

1b. Let \( u = \sin x \Rightarrow \frac{du}{dx} = \cos x \). Then by the Fundamental Theorem and the Chain Rule we have
\[
\frac{d}{dx} \int_0^{\sin x} t^2 \, dt = \left( \frac{d}{du} \int_0^u t^2 \, dt \right) \frac{du}{dx} = u^2 \frac{du}{dx} = \sin^2 x \cos x.
\]

2a. Let \( u = 1 + x^3 \Rightarrow du = 3x^2 \, dx \), so
\[
\int_0^2 3x^2 \sqrt{1 + x^3} \, dx = \int_1^9 \sqrt{u} \, du = \left[ \frac{2}{3} u^{3/2} \right]_1^9 = \frac{2}{3} (9^{3/2} - 1^{3/2}) = \frac{2}{3} (27 - 1) = \frac{52}{3}.
\]

2b. Let \( u = \sin x \Rightarrow du = \cos x \, dx \), so
\[
\int_0^{\pi/2} \cos x \cos (\sin x) \, dx = \int_0^1 \cos u \, du = [\sin u]_0^1 = \sin 1.
\]

3. The error formula for the trapezoidal rule is \( |E_T| \leq \frac{b-a}{12} h^2 M \) where \( M \) is any upper bound on \( |f''(x)| \). In this problem we have \( b = 2, a = 0, h = \frac{b-a}{n} = \frac{2}{n} \), and \( f''(x) = 6x + 4 \). Therefore, since \( |f''(x)| \leq 6 \cdot 2 + 4 = 16 \) on \([0, 2]\) we have \( M = 16 \). In order to make \( |E_T| < \frac{1}{100} \), we need
\[
\frac{b-a}{12} h^2 M = \frac{2}{12} \left( \frac{2}{n} \right)^2 \cdot 16 < \frac{1}{100},
\]
\[
\frac{2^5}{3n^2} < \frac{1}{22 \cdot 5^2},
\]
\[
n^2 > \frac{3}{2^7 \cdot 5^2},
\]
\[
n > \sqrt{\frac{3}{2^7 \cdot 5^2}}
\]
\[
n > 2^3 \cdot 5 \sqrt{\frac{2}{3}} = 40 \sqrt{\frac{2}{3}}.
\]

Since \( \sqrt{\frac{2}{3}} < 1 \), we can choose \( n = 40 \) to make the error less than \( \frac{1}{100} \).

4a. By definition, \( \text{av}(f) = \frac{1}{b-a} \int_a^b f(x) \, dx \). Therefore, \( \text{av}(f) \) over \([0, 2]\) is given by \( \frac{1}{2} \int_0^2 f(x) \, dx \). Using one of our rules for definite integrals we have \( \int_0^2 f(x) \, dx = \int_0^1 f(x) \, dx + \int_1^2 f(x) \, dx \). Since \( \text{av}(f) \) over \([0, 1]\) is 8 we have \( \int_0^1 f(x) \, dx = (1 - 0) \cdot 8 = 8 \) and since \( \text{av}(f) \) over \([0, 2]\) is 4 we have \( \int_1^2 f(x) \, dx = (2 - 1) \cdot 4 = 4 \). Therefore, \( \text{av}(f) \) over \([0, 2]\) is \( \text{av}(f) = \frac{1}{4} (8 + 4) = 6 \).

4b. To get \( \text{av}(f) \) over \([3, 6]\) we use the same reasoning as above. By definition, \( \text{av}(f) = \frac{1}{b-a} \int_a^b f(x) \, dx = \frac{1}{3} \int_3^6 f(x) \, dx \). Since \( \text{av}(f) \) over \([3, 4]\) is 3 we have \( \int_3^4 f(x) \, dx = (4 - 3) \cdot 3 = 3 \) and since \( \text{av}(f) \) over \([4, 6]\) is 9 we have \( \int_4^6 f(x) \, dx = (6 - 4) \cdot 9 = 18 \). Therefore, \( \text{av}(f) \) on \([3, 6]\) is \( \text{av}(f) = \frac{1}{3} \left( \int_3^4 f(x) \, dx + \int_4^6 f(x) \, dx \right) = \frac{1}{3} (3 + 18) = 7 \).

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5. The area of each cross section is given by \( A(x) = \frac{1}{2} [s(x)]^2 \) where \( s(x) = (2 - x^2) - (-6 + x^2) = 8 - 2x^2 \). The parabolas intersect at \( x = \pm 2 \) so the volume is given by:

\[
\text{volume} = \int_{-2}^{2} A(x) \, dx = \int_{-2}^{2} \frac{1}{2} (8 - 2x^2)^2 \, dx,
\]

\[
= \frac{1}{2} \int_{-2}^{2} 64 - 32x^2 + 4x^4 \, dx = \frac{1}{2} \left[ 64x - \frac{32}{3} x^3 + \frac{4}{5} x^5 \right]_{-2}^{2} = 1024 \frac{15}{15}.
\]

6. To find the volume we use washers. The inner radii are given by \( r(y) = \sin y \) and the outer radii are given by \( R(y) = 2 - \cos y \). Therefore, the volume is:

\[
\text{volume} = \int_0^\pi \pi [R(y)^2 - r(y)^2] \, dy = \int_0^\pi \pi [(2 - \cos y)^2 - (\sin y)^2] \, dy,
\]

\[
= \pi \int_0^\pi 4 - 4 \cos y + \cos^2 y - \sin^2 y \, dy = \pi \int_0^\pi 4 - 4 \cos y + \cos 2y \, dy,
\]

\[
= \pi \left[ 4y - 4 \sin y + \frac{1}{2} \sin 2y \right]_0^\pi = 4\pi^2.
\]

7. To find the volume we use cylindrical shells. The heights of the shells are given by \( f(x) = 12(x^2 - x^3) \). Therefore, the volume is:

\[
\text{volume} = \int_0^1 2\pi x f(x) \, dx = \int_0^1 2\pi x [12(x^2 - x^3)] \, dx,
\]

\[
= 24\pi \int_0^1 x^3 - x^4 \, dx = 24\pi \left[ \frac{1}{4} x^4 - \frac{1}{5} x^5 \right]_0^1 = \frac{6\pi}{5}.
\]