Representation Theory via Counting Polytopes

There is a long history of combinatorics in the representation theory of the general linear groups, e.g., computing dimensions of representations by counting semistandard Young tableaux. In '95-'01, Ginzburg, Lusztig, and Mirkovic-Vilonen developed geometric representation theory to handle (finite-dimensional, reductive) Lie groups in a uniform way. J. Anderson and Kamnitzer distilled uniform combinatorial rules for these representation theory numbers (weight multiplicities, tensor product multiplicities) by counting certain polytopes, now admitting a high-school-level description. I'll survey these results, and talk about recent work of mine with Kamnitzer connecting these results to a completely different sort of geometric representation theory.