On Ramanujan Graphs and Ramanujan Complexes

The renowned Ramanujan conjecture, proved by Deligne, estimates the Fourier coefficients of cusp forms. Its extension to automorphic forms for $GL_n$ over a function field is established by Drinfeld and Lafforgue. Initiated by the work of Lubotzky-Phillips-Sarnak and, independently, Margulis, these facts can be used to construct spectrally optimal combinatorial objects, called Ramanujan graphs and complexes.

In this talk we shall review the development of Ramanujan graphs and complexes, and introduce a new way to characterize them in terms of the associated zeta functions satisfying the Riemann Hypothesis.