The inverse Wronskian problem is to find, for each polynomial \( h(z) \), all vector spaces that are spanned by polynomials with Wronskian equal to \( h(z) \). In this talk, I will consider a variant on this problem, in which one is interested only in those vector spaces that are invariant subspaces of a Möbius transformation. The most basic question one can ask is how many are there?

At first glance, this may seem like a contrived twist on a contrived problem. The main goal of my talk will be to convince you that this is far from the case. Rather, this problem turns out to have some surprising connections to other topics, including control theory, real algebraic curves, real Schubert calculus, and the cyclic sieving phenomenon in enumerative combinatorics. Along the way, I’ll give two different answers to our basic question, and discuss some of their implications.