Extra Weekend Problems 1

June 30, 2007

1. Compute the differentials of these functions at the given value of $x$:

(a) $x^2 + 3x + 1$ at $x = 2$.
(b) $x^4 + 2x + \sin(x)$ at $x = 0$.
(c) $\frac{1}{1-x}$ at $x = 0$.
(d) $\frac{(1-x)^2}{1+x}$ at $x = 0$.
(e) $\tan(x)$ at $x = 0$. (remember how to define tan in terms of cos and sin)
(f) $\cos^2(x) - \sin^2(x)$ at $x = \pi/2$.
(g) $1$ at $x = 7$.
(h) $\cos(\frac{\pi}{2} + x)$ at $x = 0$.
(i) $e^{(x+1)^2}/e^{x+1}$ at $x = 0$.
(j) $x^2e^x \cos(x) \sin(x)$ at $x = \pi$.

2. Use differentials to compute the derivative of these functions at the given value of $x$:

(a) $x^2e^{2x}$ at $x = 2$.
(b) $\cos(x^2 - x)$ at $x = 0$.
(c) $e^{\cos(x) + \sin(x)}$ at $x = 0$.
(d) $x \tan(x)$ at $x = \pi$.
(e) $e^{(x+1)^2}$ at $x = 1$.
(f) $e^{(\cos(x) + \sin(x))^2}$ at $x = 0$.
(g) $e^x$ at $x = 0$.

3. Assume that $f$ is differentiable and use the given information to write the equation of a tangent line to $f$:

(a) $f(1 + dx) = -1 - dx$.
(b) $f(1 - dx) = -1 - dx$. 


(c) \( f(2 + dx) = 2 + 3dx \).
(d) \( f(3 + 2dx) = 2 + 3dx \).

4. Using only the derivative rules that we have already learned (for \( x^n \), \( e^x \), \( \cos(x) \) and \( \sin(x) \)) and the formula \( f(x + n \, dx) = f(x) + f'(x) \cdot n \, dx \) that we used in class on Friday, compute the derivative of each function (don’t use the chain rule or product rule if you know what they are, just direct computation!):

(a) \( x^{17} \)
(b) \( 3x^3 + 2x^2 + x \)
(c) \( e^x + x^2 - \cos(x) \)
(d) \( e^{x^2+1} \)
(e) \( e^{-x} \)
(f) \( \cos(e^x) \)
(g) \( e^{\cos(x)} \)
(h) \( \cos(x) \sin(x) e^x \)
(i) \( 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} + \ldots \)
(j) \( \frac{1}{\cos(x)} \). You might find the formula

\[
\frac{1}{a-b} = \frac{1}{a} \cdot \left(1 + \frac{b}{a} + \left(\frac{b}{a}\right)^2 + \left(\frac{b}{a}\right)^3 + \ldots\right)
\]

useful. This formula comes from short manipulation of the formula

\[
\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots
\]

which we have already used before.

5. Show that \( x^3 + x^2 - 3x + 1 \) is continuous.

6. Show that \( e^{\cos(x)} \) is continuous.

7. Show that \( \cos(e^x) \) is continuous.