I have been teaching math nearly as long as I have been studying math. Throughout college and graduate school I have had the opportunity to teach a wide variety of classes, from SAT courses for the Princeton Review to calculus, writing, and even undergraduate research supervision. In this note, I would like to describe several common threads which I have followed through these teaching experiences.

Classroom Philosophy

As an undergraduate at an experimental liberal arts college, I rapidly realized how much better I learned when I was invested in a course’s material. My first encounter with serious mathematics came from my attempt to learn enough field theory to understand how error correcting codes are constructed. I have tried to bring the power of such an investment to each class I have taught. I believe that to succeed in mathematics, students need at least three things:

- The ability to see math as a descriptive language for abstract ideas.
- The ability to interact and freely play with mathematical objects.
- The desire and opportunity to do so!

As an example of how the first point is realized, I usually spend the beginning of an introductory calculus course discussing calculus as a language\(^\text{1}\) before we learn it as a computational tool. A typical problem assigned after the first day (where the symbols ‘\(\,\)’ and ‘\(\int\)’ are introduced to mean “change in” and “total change”, respectively) might be: “Find several newspaper headlines describing different types of change and translate them into equations”.

Supporting the second and third goals, I try to draw student’s interest through mathematical “play”: small problems which an actual mathematician might find interesting. As one particular example, in a Calculus 1 course I have sometimes asked the students to think about how they would attempt to compute the derivative of the exponential tower

\[
x^{x^{x^{\ldots}}}
\]

By including problems like this as challenging (and often ungraded) exercises, I try to encourage students to really play with the tools they have learned. Often, this involves small-group work in the classroom; the students within each group provide a source of ideas, which can then be played with and kept, discarded, or set aside for consideration by the whole class. I’ve found days where we did such group work to be some of the most productive of the semester.

One very nice payoff from these two approaches is that once we get to typically difficult first-year calculus problems such as related rates, the students seem to be better at modeling and reasoning about problems more abstractly. This dramatically improves their ability to solve the dreaded “word problems” (which of course are the most interesting problems of all!)

\(^{1}\)I was influenced in this direction by the novel calculus textbook *Calculus: the Language of Change*, written by Jim Henle and David Cohen. Though I have been teaching from more standard textbooks, I find that a periodic reminder of the linguistic nature of calculus helps the students make sense of all the techniques they are learning.
In a somewhat different application of this philosophy, I based my writing course in 2007 on the notion that a student who cares deeply about a particular mathematical idea will naturally want to communicate it to a non-mathematical audience of peers. In this way, the students were motivated to acquire the tools and the clarity of thought needed to properly communicate their ideas to others through writing.

**Novel Courses**

I feel lucky that I have had several opportunities to try novel teaching methods. The two most notable cases are the writing seminar which I mentioned above, and an introductory calculus class in summer 2007 which began with infinitesimals rather than limits. There are obviously pros and cons to such a method, but I do not have the space in this short letter to make an adequate case for this approach. Instead, I will simply note one objective effect: using infinitesimals instead of limits reduces the \( \Pi_n \) complexity of logical sentences (measured by number of quantifier alternations) by an entire degree. To see why this could aid in comprehension, compare the following \( \Pi_1 \) sentence:

\[
\forall \varepsilon > 0, x, y \exists \delta > 0 : |x - y| < \delta \implies |f(x) - f(y)| < \varepsilon
\]

(which we expect any calculus 1 student to understand) versus the \( \Pi_2 \) sentence:

\[
\forall \varepsilon > 0 \exists \delta > 0 \forall x, y : |x - y| < \delta \implies |f(x) - f(y)| < \varepsilon
\]

(the understanding of which we usually reserve for majors in an analysis course). Using infinitesimals, the first sentence can be replaced with the \( \Pi_0 \) sentence:

\[
\forall x, y : x \approx y \implies f(x) \approx f(y)
\]

which is notable for how closely it symbolically mirrors our intuitive notion of continuity.

I would be happy to provide more information about either of these courses by email or in person. You can also find syllabi, homework assignments, and tests from these courses at my website [http://www.math.cornell.edu/~noonan](http://www.math.cornell.edu/~noonan).

**Undergraduate Research**

Between attending a project-based college and my experiences being in and helping to run various REU and REU-style programs, I have had a lot of exposure to undergraduate research. My general teaching philosophy is already hands-on and experimental; I think this translates well to undergraduate research. For example, when I ran the research projects for the 2009 SMI program, I chose three general areas of inquiry (the Loewner differential equation, minimal surfaces, and structure types in combinatorics) which are generally rather abstract. But in each of these cases I had particular experiments in mind which helped the students immediately begin to engage with the chosen subject area. To take one group’s experience, the students working on the Loewner equation wanted to investigate a how a curve called the trace varied as they changed a certain driving function. To do this, they created several programs using a variety of techniques (which they invented or re-invented!) for computing the trace. This gave the students a real grounding in their subject area — when later investigations became more abstract, they often fell back on these early experiments to help understand what was going on.
Technology

I have somewhat contradictory feelings about technology in the classroom. On the one hand, I much prefer to use chalk while giving a lecture, perhaps with the occasional use of the computer to run an experiment or visualization. I find that by primarily interacting with a physical medium, I have an easier time keeping the pace of a lecture at a manageable level and responding flexibly to the needs of the students.

Despite my personal preference for the “chalk-talk”, I think that facility with a computer (and programming ability in particular) provides a substantial advantage when learning math. I know that personally, I have sometimes found it too easy to forget that my calculations and abstractions correspond to “real” things. Often the only way to interact or experiment with these “real” things is through the computer.

To this end, I am very interested in teaching or developing a course in programming specifically aimed at undergraduate math majors, though I have not had the chance to teach such a class at Cornell. Natural motivating topics include dynamical systems, group theory, graph theory, differential equations, algebraic coding, or complexity theory. A course centered around Python and the free mathematical software suite SAGE could also be interesting. It is even possible to combine basic programming with basic calculus. The calculus sequence at Hampshire College\(^2\) follows this approach; the actual act of programming an ODE solver and a Riemann integrator yields a very hands-on proof of the fundamental theorem of calculus!

\(^2\)This particular sequence was the motivator for the Five College Calculus in Context program. A short excerpt from their “creation story”: *The core of the course was calculus, but calculus as it is used in contemporary science. Mathematical ideas and techniques grew out of scientific questions. Given a process, students had to recast it as a model; most often, the model was a set of differential equations. To solve the differential equations, they used numerical methods implemented on a computer.*