A Note on Infinite Series

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Math 311

A note about series

- Recall that a series is an "infinite sum"

$$\sum_{n=1}^{\infty} a_n.$$

- A series is **absolutely convergent (AC)** if the series

$$\sum_{n=1}^{\infty} |a_n|$$

converges.

- An absolutely convergent series is also convergent, in the sense that $\sum a_n$ converges as well.

- Examples of AC series are

$$\sum \frac{(-1)^n}{n^2}, \sum \frac{1}{n^2}, \sum \frac{1}{n!}, \dots$$

- A series is **conditionally convergent (CC)** if it is convergent but not absolutely convergent.

- Note that that a series can be convergent and fail to be AC (that is, CC) only if it contains negative terms as well; the most common examples are the alternating series.

- An example of a CC series

$$\sum \frac{(-1)^n}{n}.$$

Test for convergence

- The idea is that you first want to test for AC using the ratio test or the nth root test. This test ise for AC or divergence (D).

- If this test is inconclusive (the corresponding limit equals 1), than you should apply the comparison test, or the integral test to the series

$$\sum |a_n|.$$

- If this series is convergent, then the original series is AC.

- If this series is divergent, but the original series is convergent (using the Alternating series or Cauchy test, for example), then the series is CC.