We define V_m to be the network derived from the mth level graph of the Sierpinski Gasket by performing Δ-Y transformations on each m-cell. Let R_0, R_1, R_2 be the resistances on the edges of V_0 after restricting a graph V_m with edge resistances 1 except on a cell F_w where $|w| = m$ and the edges have resistance $1+t$. We would like to calculate their derivatives respectively with respect to t. To do this, we come up with a general algorithm of determining the derivatives given some conditions. We look first at a level graph where the edges have resistance one except for the top cell which has resistances $1+at+O(t^2), 1+bt+O(t^2), 1+ct+O(t^2)$ (we omit the $O(t^2)$ in the writing, but remind the reader that the functions, though we only write the first two terms of their Taylor expansions, are rarely linear, and these are the expansions of rational functions). Using edge addition and the Δ-Y algorithm, and then expanding the resistances by Taylor expansion, as shown in figire 2, we find that

$$R_1'(0) = 1 + \frac{2}{9}(b+c),\ R_2'(0) = \frac{1}{9}(2b-c),\ \text{and}\ R_3'(0) = \frac{1}{9}(2c-b).$$

Suppose further that this V_1 cell was actually a subcell of a larger V_2 cell. Then we reduce each V_1 cell independently and we see that the other two V_1 cells, when rediced to V_0 cells, all have resistances $\frac{5}{3}$, and our cell in question is reduced as we did before. If we reduce again, we get figure 3, and we can perform the same operation again on the coefficients of the resistances, letting $1+\frac{2}{9}(b+c) = a'$, $\frac{2}{9}(2b-c) = b'$, and $\frac{2}{9}(2c-b) = c'$. Also note that the transformation of the derivatives a, b, c was linear, given by the matrix

$$D_0 = \begin{bmatrix} 1 & \frac{2}{9} & \frac{2}{9} \\ 0 & \frac{2}{9} & -\frac{1}{9} \\ 0 & -\frac{1}{9} & \frac{2}{9} \end{bmatrix}.$$

In general, if we have a V_1 graph with resistances x^n except in the top cell where x^n+at, x^n+bt, x^n+ct, where $x=\frac{5}{3}$, then the reduced Y graph will have resistances $x^{n+1} + \left(a + \frac{2}{9}(b+c)\right), x^{n+1} + \frac{1}{9}(2b-c)$, and $x^{n+1} + \frac{1}{9}(2c-b)$. Moreover, we can generalize the case to when the nonconstant resistances are in the second or third cells of a V_1 graph, in which we just rotate the graph and apply the same transformation as before, where the matrices on the derivatives a, b, c will be

$$D_1 = \begin{bmatrix} \frac{2}{9} & 0 & -\frac{1}{9} \\ \frac{2}{9} & 1 & \frac{2}{9} \\ -\frac{1}{9} & 0 & \frac{2}{9} \end{bmatrix}, D_2 = \begin{bmatrix} \frac{2}{9} & -\frac{1}{9} & 0 \\ -\frac{1}{9} & \frac{2}{9} & 0 \\ \frac{2}{9} & \frac{2}{9} & 1 \end{bmatrix}$$

where D_i is applied if the nonconstant edges are in the F_i cell. Using these matrices, if we perturb a cell F_w with $|w| = m$ in a graph V_m, then the derivatives of the resistances R_i in the reduced Y graph will be

$$e_i^T D_w \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{1}$$

respectively, where

$$D_w = \prod_{i=1}^{m} D_{w_i}.$$

Figure 1

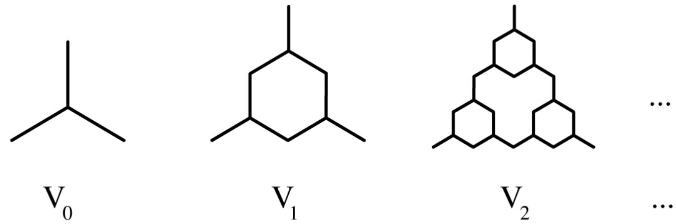

Figure 2

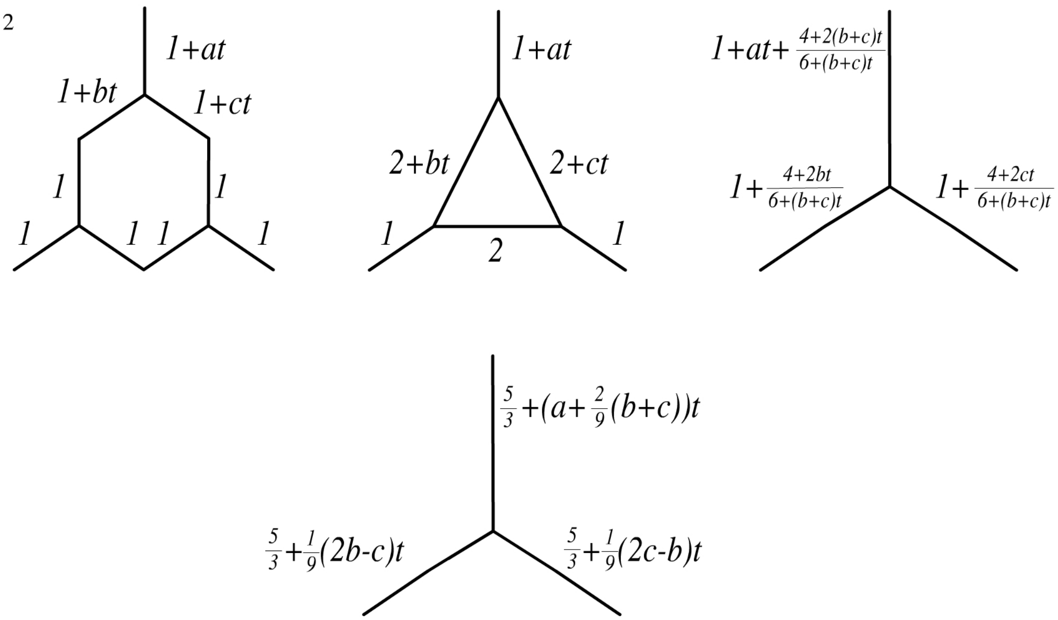

Figure 3