Consider the following matrices.

$$A := \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B := \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad C := \begin{pmatrix} 5 & 5 & 1 \\ 0 & 3 & 6 \\ 0 & 0 & 2 \end{pmatrix}$$

$$D := \begin{pmatrix} 3 & 0 & 0 \\ 3 & 8 & 0 \\ 4 & 7 & 7 \end{pmatrix}, \quad E := \begin{pmatrix} 2 & 3 & 3 & 1 \\ 0 & 1 & 7 & 6 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \quad F := \begin{pmatrix} 8 & 3 & 3 & 5 \\ 1 & 2 & 5 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

For each matrix M below, do the following:

1. Find the characteristic polynomial χ_M of M.
2. Find the eigenvalues of M.
3. Find all eigenvectors of M.
4. Determine if M is diagonalizable, and if so find matrices P and D such that $M = PDP^{-1}$.
5. Solve the differentiable equation $x' = Mx$.
6. Determine if the columns of M form an orthogonal (likewise, orthonormal) set.