

**Question 1:**

Consider the system of equations:

$$x_1 + 3x_2 + 3x_3 - 2x_4 = 13$$

$$-2x_1 + x_2 - x_3 + 2x_4 = -2$$

$$-x_1 - 4x_2 - 2x_4 = 17$$

$$-x_1 - x_2 - 4x_3 + 5x_4 = 34$$

Write down the augmented matrix for this system.

*Solution:*

$$\left[ \begin{array}{cccc|c} 1 & 3 & 3 & -2 & 13 \\ -2 & 1 & -1 & 2 & -2 \\ -1 & -4 & 0 & -2 & 17 \\ -1 & -1 & -4 & 5 & 34 \end{array} \right]$$

**Question 2:**

Let  $M = \begin{bmatrix} 1 & 4 & -3 & -3 & -27 \\ 1 & 5 & -1 & -4 & -22 \\ -3 & 1 & 5 & -4 & 26 \end{bmatrix}$ . Reduce  $M$  to echelon form. Label every step in the reduction.

*Solution:*

Subtract row 1 from row 2:

$$\left[ \begin{array}{ccccc} 1 & 4 & -3 & -3 & -27 \\ 0 & 1 & 2 & -1 & 5 \\ -3 & 1 & 5 & -4 & 26 \end{array} \right]$$

Add 3 \* row 1 to row 3:

$$\left[ \begin{array}{ccccc} 1 & 4 & -3 & -3 & -27 \\ 0 & 1 & 2 & -1 & 5 \\ 0 & 13 & -4 & -13 & -55 \end{array} \right]$$

Subtract 13\*row 2 from row 3:

$$\left[ \begin{array}{ccccc} 1 & 4 & -3 & -3 & -27 \\ 0 & 1 & 2 & -1 & 5 \\ 0 & 0 & -30 & 0 & -120 \end{array} \right]$$

**Question 3:** Suppose that  $M$  is the augmented matrix for some linear system. How many solutions does it have?

*Solution:* There's no pivot in the last column, so the system is consistent. There's no pivot in column 4, so there's a free variable. Hence the system has infinitely many solutions.