Mosquito Bites

The Zika virus is one of the most feared infectious diseases of our day. The illness is usually mild with symptoms lasting about a week, but if it occurs during a pregnancy, it often leads to serious birth defects (e.g., microcephaly). This disease is spread primarily through bites of Zika-infected female mosquitoes. As of right now, there is no vaccine for Zika and all prevention techniques focus on
  a) reducing the chances of mosquito bites through human behavioral measures (e.g., travel restrictions, insect repellants, long-sleeved shirts, bed nets, etc);
  b) reducing the local population of mosquitoes (e.g., use of insecticides, eliminating breeding grounds);
  c) reducing mosquitoes’ capacity to transmit Zika to humans.
Both (b) and (c) can be accomplished by introducing into the environment some “modified mosquitoes”. This is precisely the plan currently considered in Puerto Rico and your team was hired to evaluate its feasibility.

In particular, the plan is to grow in the laboratory and release into the environment mosquitoes carrying the Wolbachia bacterium. Female mosquitoes of this type (fW) are far less likely to spread Zika. Moreover, “regular” female mosquitoes (fR) lay eggs that do not hatch after mating with Wolbachia-carrying male mosquitoes (mW). In contrast all (fW + mR) matings produce Wolbachia-carrying offspring. (See the diagram at www.eliminatedengue.com/library/feature/ft/CI-Wolbachia-spread-diagram-large.jpg)

So, Puerto Rico is considering two different options:
  1) Introducing only mW’s with the goal of (temporarily) reducing the overall mosquito population.
  2) Introducing both mW’s and fW’s, with the goal of eventually replacing most “regular” mosquitoes by Wolbachia-carrying mosquitoes that are incapable of transmitting Zika.
Of course, you can also recommend a time-dependent strategy, switching between the above options. Your plan should cover 5 years and guarantee that
  A. The total number of Zika-infected pregnant women in Puerto Rico over these 5 years should not exceed 10000. (Note: based on CDC statistics, as of right now, about 22 pregnant women get newly Zika-infected in Puerto Rico every day.)
  B. After the first year, the number of newly infected people per day should not exceed 300. (Note: as of right now, the number of newly infected Puerto Ricans is estimated to be 600 per day, including the 80% of infections that are asymptomatic and usually go unreported.)

Suppose you are allowed to release at most $K$ Wolbachia-carrying mosquitoes per month. What is the smallest $K$ that would allow you to meet the above constraints? How would you vary the newly released fW/mW ratio over time? What would be the total number of mosquitoes released over 5 years under your plan?
You may assume that 1% of fR's are currently Zika-infected and that all new cases of human infection are local (e.g., no one gets bitten by a Zika-infected mosquito while traveling outside of Puerto Rico). You may also assume that migration of mosquitoes into and out of Puerto Rico is small enough and can be neglected.

Many people are understandably alarmed by the plan to release even more mosquitoes in their country. Write a short “Letter to the Editor” to explain the advantages & limitations of your approach to the general public.