

**Question 1**

**mode** MultipleChoice

**text**

A reduced row-echelon form of a 3 by 4 matrix can have how many leading one's?

**choice** must have 3

**choice** may have 1, 2, or 3

**correct-choice** may have 0, 1, 2, or 3

**choice** may have 0, 1, 2, 3, or 4

**choice** must have 4

**Question 1**

**mode** MultipleChoice

**text**

Consider the following statements about a 3 by 3 matrix  $X$ :

A. The rank of  $X$  equals 2.

B. The rank of  $X$  is  $\leq 2$ .

C. One row of  $X$  is a multiple of another row.

Which of the following is correct?

**choice** A implies C, but C does not imply A

**choice** B implies C, but B does not imply A

**choice** C implies A, but A does not imply C

**choice** C implies A and A implies C

**correct-choice** C implies B, but B does not imply C

**choice** C implies B and B implies C

**choice** C does not imply B, and B does not imply C

**Question 1**

**mode** MultipleChoice

**text** Suppose  $A$  is a  $3 \times 3$  matrix defining a linear transformation  $T$  from  $R^3$  to  $R^3$ . such that

$$T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \dots \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \text{ Which statement is correct?}$$

**choice** The first row of  $A$  is  $(1 \ 0 \ 0)$

**choice** At least one row of  $A$  is  $(1 \ 0 \ 0)$  but we don't know which one

**choice** All rows of  $A$  are  $(1 \ 0 \ 0)$

**correct-choice** The first column of  $A$  is  $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

**choice** At least one column of  $A$  is  $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$  but we don't know which one

**choice** All columns of  $A$  are  $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

**choice** None of the above and below

**Question 2**

**mode** MultipleChoice

**text** Let  $A = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$ . Which of the following is the correct geometric interpretation of the associated linear transformation?

**choice** rotates counterclockwise through 180 degrees

**correct-choice** rotates counterclockwise through 90 degrees and doubles the length

**choice** rotates counterclockwise through 90 degrees and halves the length

**choice** rotates counterclockwise through 90 degrees and quadruples the length

**choice** rotates clockwise through 90 degrees and doubles the length

**Question 1**

**mode** MultipleChoice

**text** Let  $T$  be a linear transformation from  $R^n$  to  $R^m$  and let  $\vec{u} = T(\vec{0})$  where  $\vec{0}$  is the zero vector in  $R^n$ . Choose the correct statement

**choice**  $\vec{u}$  is a zero vector in  $R^n$

**choice**  $\vec{u}$  is a zero vector in  $R^m$  if and only if  $n \leq m$

**choice**  $\vec{u}$  is a zero vector in  $R^m$  if and only if  $n = m$

**choice**  $\vec{u}$  is a zero vector in  $R^m$  if and only if  $n \geq m$

**correct-choice**  $\vec{u}$  is a zero vector in  $R^m$

**choice** none of the above and below

**Question 2**

**mode** MultipleChoice

**text** Let  $T$  be a rotation (with matrix  $A$ ) through angle  $\alpha$  in the plane. Choose the correct statement about the columns of  $A$  (as vectors)

**correct-choice** The columns of  $A$  are of length one and the dot product of the two distinct columns equals zero

**choice** The columns of  $A$  are of length one and the dot product of the two distinct columns equals  $\cos(\alpha)$

**choice** The columns of  $A$  have length  $\cos^2(\alpha)$

**choice** The columns of  $A$  have length greater than one

**choice** The dot product of the two distinct columns is always positive

**choice** The dot product of the two distinct columns is zero if and only if  $\alpha = 0$

**choice** None of the above and below

**Question 1**

**mode** MultipleChoice

**text** Suppose  $T: R^n \rightarrow R^m$  is a linear transformation and there exists a vector  $\vec{v}$  (not zero) such that  $T\vec{v} = \vec{v}$ . Then

**choice** T might be a rotation, shear, reflection or a projection

**choice** T might be a reflection or projection, but not a rotation or shear

**correct-choice** T might be a reflection, projection or shear but not a rotation

**choice** T might be a reflection, shear or rotation but not a projection

**choice** T might be a projection, shear or rotation but not a reflection

**choice** none of the above and below

**Question 2**

**mode** MultipleChoice

**text** Let  $T = proj_L$  and let  $\vec{u}$  be a vector of length 1. Then the length of  $T\vec{u}$  might be:

**choice** any positive number ( $x > 0$ )

**choice** any nonnegative number ( $x \geq 0$ )

**choice** any x satisfying  $0 < x < 1$

**correct-choice** any x satisfying  $0 \leq x \leq 1$

**choice** either 0 or 1

**choice** must be 1

**Question 1**

**mode** MultipleChoice

**text** The inverse of the matrix  $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$  is

**choice**  $\begin{pmatrix} 3 & -1 \\ -2 & 4 \end{pmatrix}$

**choice**  $\begin{pmatrix} -4 & 2 \\ 1 & -3 \end{pmatrix}$

**choice**  $\begin{pmatrix} \frac{-4}{10} & \frac{2}{10} \\ \frac{1}{10} & \frac{-3}{10} \end{pmatrix}$

**correct-choice**  $\begin{pmatrix} \frac{3}{10} & \frac{-1}{10} \\ \frac{-2}{10} & \frac{4}{10} \end{pmatrix}$

**choice** the matrix is not invertible

**Question 2**

**mode** MultipleChoice

**text** Suppose you need to solve a system of 4 equations in 4 unknowns  $A\vec{x} = \vec{b}$  for a specific 4\*4 matrix A and a specific 4-vector  $\vec{b}$ . You now know two methods:

(i) use Gauss-Jordan elimination on the augmented matrix  $(A|\vec{b})$

(ii) find  $A^{-1}$  and then set  $x = A^{-1}\vec{b}$

Choose the best statement comparing the relative amount of work that these methods require:

**choice** Both require the same amount of work

**choice** (ii) is easier because there is a formula for  $A^{-1}$

**choice** (ii) is harder because computing  $A^{-1}$  is just as hard as (i) but then you have to do the matrix multiplication  $A^{-1}\vec{b}$

**correct-choice** (ii) is harder because computing  $A^{-1}$  involves row reducing a larger matrix, and in addition you have to do the matrix multiplication

**choice** it depends on what A is

**Question 1**

**mode** MultipleChoice

**text** Let  $B = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \end{pmatrix}$  and  $A = \begin{pmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{pmatrix}$ .... Then  $BA = ..?$

**choice**  $\begin{pmatrix} 0 & 12 \\ 0 & 12 \end{pmatrix}$

**correct-choice**  $\begin{pmatrix} 0 & 0 \\ 12 & 12 \end{pmatrix}$

**choice**  $\begin{pmatrix} 6 & 4 & 2 \\ 6 & 4 & 2 \\ 6 & 4 & 2 \end{pmatrix}$

**choice**  $\begin{pmatrix} 6 & 6 & 6 \\ 4 & 4 & 4 \\ 2 & 2 & 2 \end{pmatrix}$

**choice** undefined

**Question 2**

**mode** MultipleChoice

**text** Let  $L_1$  and  $L_2$  be two distinct lines passing through the origin (of  $R^2$ ), and let  $ref_{L_1}$  and  $ref_{L_2}$  denote the reflection inlines  $L_1$  and  $L_2$  respectively. What kind of linear transformation is the composition  $ref_{L_1}(ref_{L_2}(\vec{x}))$ ? (Suggestion: use geometry rather than algebra)

**choice** A reflection in another line

**choice** the identity

**correct-choice** a rotation

**choice** a shear

**choice** a projection

**choice** a dilation

**choice** a linear transformation, but not of the types listed here

**choice** not a linear transformation

**Question 1**

**mode** MultipleChoice

**text** Let A and B be invertible  $n * n$  matrices. Then the inverse of  $ABA^{-1}$  is

**choice**  $A^{-1}B^{-1}A$

**correct-choice**  $AB^{-1}A^{-1}$

**choice**  $B^{-1}$

**choice**  $ABA^{-1}$

**choice**  $A^{-1}BA$

**choice** the matrix is not necessarily invertible

**Question 2**

**mode** MultipleChoice

**text** Consider the following three statements about an invertible  $n * n$  matrix A.

(i)  $A = A^{-1}$

(ii)  $AA = I$

(iii)  $A^{-1}A^{-1} = I$

where I is an identity  $n * n$  matrix. Choose the correct relationship among them:

**choice** (i) and (ii) are equivalent, but (iii) is different

**choice** (i) and (iii) are equivalent, but (ii) is different

**choice** (ii) and (iii) are equivalent, but (i) is different

**choice** all three statements are different

**correct-choice** all three statements are equivalent



### Question 1

**mode** MultipleChoice

**text** Let  $T$  be the orthonormal projection in the plane onto the line (through the origin)  $L$ . Fill in the blank with a, b, c, d or e:  
The image of  $T$  is:

**correct-choice** the line  $L$

**choice** the origin

**choice** the whole plane

**choice** the line (through the origin) orthogonal to  $L$

**choice** a line parralel to  $L$

### Question 2

**mode** MultipleChoice

**text** Let  $T$  be the orthonormal projection in the plane onto the line (through the origin)  $L$ . Fill in the blank with a, b, c, d or e:  
The kernel of  $T$  is

**choice** the line  $L$

**choice** the origin

**choice** the whole plane

**correct-choice** the line (through the origin) orthogonal to  $L$

**choice** a line parralel to  $L$

### Question 3

**mode** MultipleChoice

**text** The kernel of  $\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix}$  is

**choice**  $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

**choice**  $\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

**choice** the line  $x + 2y + 3z = 0$  in  $R^3$

**choice** the plane  $x + 2y + 3z = 0$  in  $R^3$

**correct-choice** the line  $x + 2y = 0$  in  $R^2$

**choice** the line  $y = 2x$  in  $R^2$

**choice** all of  $R^2$

**Question 1**

**mode** MultipleChoice

**text** Let  $V$  be a subspace of  $R^n$  and  $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$  a set of vectors in  $V$ . Suppose every vector in  $V$  may be expressed as a linear combination of  $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$ . Then

**choice**  $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$  is a basis of  $V$

**choice**  $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$  are linearly independent

**correct-choice**  $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$  span  $V$

**choice**  $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$  span  $R^n$

**choice**  $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$  are linearly dependent

**choice** none of the above and below

**Question 2**

**mode** MultipleChoice

**text** In example 4 on page 113, which pairs of the vectors span image  $A$ ?

**choice** all pairs

**choice** only  $\{v_1, v_2\}$

**choice** only  $\{v_1, v_2\}$  or  $\{v_2, v_3\}$

**choice** only  $\{v_1, v_2\}$ ,  $\{v_2, v_3\}$  or  $\{v_3, v_4\}$

**correct-choice** all pairs except  $\{v_1, v_3\}$

**choice** all pairs except  $\{v_1, v_3\}$  and  $\{v_2, v_4\}$

**Question 1**

**mode** MultipleChoice

**text** Which one of the following statements is true of all  $m * n$  matrices A:

- correct-choice** The columns of A span  $im(A)$
- choice** The columns of A form a basis for  $im(A)$
- choice** The columns of A span  $Ker(A)$
- choice** The columns of A form a basis for  $Ker(A)$
- choice** The columns of A are linearly independent
- choice** The columns of A are linearly dependent
- choice** none of the above and below

**Question 2**

**mode** MultipleChoice

**text** Let A vary over all  $9 * 9$  matrices. How many different pairs of numbers (a,b) are there with  $a = nullity(A)$  and  $b = rank(A)$

- choice** 81
- choice** 100
- choice** 9
- correct-choice** 10
- choice** none of the above and below

**Question 1**

**mode** MultipleChoice

**text** Let  $T : R^2 \rightarrow R^2$  be the linear transformation of multiplication by the matrix  $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ . With respect to the basis  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ , the matrix representing T

is

**choice**  $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

**choice**  $\begin{pmatrix} 2 & 2 \\ 3 & -3 \end{pmatrix}$

**choice**  $\begin{pmatrix} 2 & 3 \\ 2 & -3 \end{pmatrix}$

**choice**  $\begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$

**choice**  $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

**correct-choice**  $\begin{pmatrix} \frac{5}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{pmatrix}$

**Question 2**

**mode** MultipleChoice

**text** Let A be an  $n * n$  matrix of rank m. Any matrix similar to A:

**choice** may have rank  $\leq n$

**choice** may have rank  $\leq m$

**choice** may have any rank  $\geq m$  and  $\leq n$

**correct-choice** must have rank m

**choice** may have rank m or n, but not any other rank

**Question 1**

**mode** MultipleChoice

**text** Let  $P_n$  be the linear space of all polynomials of degree  $\leq n$ . Its dimension is:

**choice** 1

**choice**  $n-1$

**correct-choice**  $n+1$

**choice**  $n^2$

**choice** infinite

**Question 2**

**mode** MultipleChoice

**text** Example 15 on page 155 gives one basis for  $P_2$ . Which of the following give another basis for  $P_2$ ?

**choice**  $(x-1)^2$ ,  $x-1$  and 1

**choice**  $(x+1)^2$ ,  $(x-1)^2$  and 1

**choice**  $x^2$ ,  $(x+1)^2$  and  $(x+2)^2$

**choice** none of these

**correct-choice** all of these

**Question 1**

**mode** MultipleChoice

**text** Let  $P_2$  be the space of quadratic polynomials. Which of the following are linear transformations from  $P_2$  to  $P_2$ ?

**choice**  $Tf(x) = xf(x)$

**choice**  $Tf(x) = f(x)^2$

**correct-choice**  $Tf(x) = f(x + 1)$

**choice** none of these

**choice** all of these

**Question 2**

**mode** MultipleChoice

**text** In example 5 on page 162, the specific matrix  $S = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$  is used. More generally, for which  $2 \times 2$  matrix  $S$  is the same conclusion valid?

**choice** all matrices

**choice** all nonzero matrices

**correct-choice** all invertible matrices

**choice** all matrices that commute with  $A$

**choice** all matrices with integer entries

**choice** only the given matrix and its inverse

**Question 1**

**mode** MultipleChoice

**text** In definition 4.3.1, if we consider the basis  $B'$  consisting of the same vectors but listed in reverse order,  $f_n, f_{n-1}, \dots, f_2, f_1$ , what do the  $B'$ -coordinates of  $f$  look like compared to the  $B$ -coordinates of  $f$ ?

**choice** the same

**correct-choice** the column listed in the reverse order

**choice** the same numbers written in a row, rather than a column vector

**choice** it depends on which vector  $f$  is

**Question 2**

**mode** MultipleChoice

**text** In Examples 4 and 5, the matrix  $B$  of the linear transformation  $T$  is computed in two ways. Use the result to find  $im(T)$  in  $P_2$ . It is

**choice** the constants

**correct-choice**  $P_1$  (all polynomials of degree  $\leq 1$ )

**choice**  $P_2$

**choice** the span of  $x + 2x^2$  and  $2x^2$

**choice** none of the above and below

**Question 1**

**mode** MultipleChoice

**text** Let  $M$  and  $M'$  denote the matrices representing the linear transformations of  $Df = f'$  from  $P^3$  to  $P^3$  with respect to the bases  $B = (1, x, x^2, x^3)$  and  $B' = (1, x - 1, (x - 1)^2, (x - 1)^3)$  respectively. Which of the following is true?

**choice**  $M' = M - I$

**choice**  $M' = M + I$

**correct-choice**  $M' = M$

**choice**  $M' = M^{-1}$

**choice** none of the above or below

**Question 2**

**mode** MultipleChoice

**text** In Example 7 consider the transformation  $T(M) = AM - MA$  for a specific matrix  $A$ . Now suppose  $A$  is any nonzero matrix. Which of the following is true?

**choice** The nullity of  $T$  is 2 because  $T(I) = 0$  and  $T(A) = 0$ , hence the rank of  $T$  is 2.

**correct-choice** The nullity of  $T$  is at least 2, hence the rank of  $T$  is at most 2.

**choice**  $T$  is an isomorphism

**choice** The nullity of  $T$  is at most 2, hence the rank of  $T$  is at least 2.

**choice**  $T$  is linear only if  $A$  is invertible

**choice** none of the above and below



**Question 1**

**mode** MultipleChoice

**text** Let  $V$  be a subspace of  $R^n$  of dimension  $m$ . Then the dimension of  $V^\perp$  is:

**choice**  $m$

**correct-choice**  $n-m$

**choice** any number between  $n - m$  and  $n$

**choice** any number between 0 and  $n - m$

**choice** any number between 0 and  $n$

**choice** none of the above or below

**Question 2**

**mode** MultipleChoice

**text** Which of the following sets of vectors constitute an orthonormal basis in  $R^3$ ?

**choice**  $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

**choice**  $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

**choice**  $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$

**choice** all of the three choices

**correct-choice** none of the three choices

### Question 1

**mode** MultipleChoice

**text** Let  $\vec{u}$  and  $\vec{v}$  be orthonormal vectors in  $R^n$ . Then  $\|u + v\|$  is

**choice** 2

**correct-choice**  $\sqrt{2}$

**choice** 1

**choice** depends on n

**choice** any number between 0 and  $\sqrt{2}$

### Question 2

**mode** MultipleChoice

**text** Let  $\vec{x}, \vec{y}$  be two nonzero vectors in  $R^n$ . Consider the 4 angles :

$a_1 =$  angle between  $\vec{x}$  and  $\vec{y}$

$a_2 =$  angle between  $-\vec{x}$  and  $\vec{y}$

$a_3 =$  angle between  $\vec{x}$  and  $-\vec{y}$

$a_4 =$  angle between  $-\vec{x}$  and  $-\vec{y}$ .

Which of the following is true:

**correct-choice**  $a_1 = \pi + a_2 = \pi + a_3 = a_4$

**choice**  $a_1 = a_2 = a_3 = a_4$

**choice**  $a_1 = \pi - a_2 = \pi - a_3 = a_4$

**choice**  $a_1 = \pi + a_2 = a_3 = \pi + a_4$

**choice**  $a_1 = a_4$  and  $a_2 = a_3$ , but there is no relationship between  $a_1$  and  $a_2$

**choice** none of the above and below

**Question 1**

**mode** MultipleChoice

**text** In Fact 5.2.2, what is the relationship between the numbers  $m$  and  $n$ ?

**choice**  $n$  and  $m$  may be any positive integers

**choice** we must have  $n \leq m$

**correct-choice** we must have  $m \leq n$

**choice** we must have  $m = n$

**choice** we must have  $n < m$

**choice** we must have  $m < n$

**Question 2**

**mode** MultipleChoice

**text** If you try to use the Gram-Schmid process (Algorithm 5.2.1) on a set of vectors  $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$  that are linearly dependent, what will happen?

**choice** It will work exactly as in linear independent case

**choice** It will produce an orthonormal, basis but with fewer than  $m$  vectors

**correct-choice** For some  $j$  the formula for  $w_j$  will call for division by zero

**choice** It will produce a set of vectors that aren't orthogonal

**choice** You can't predict what will happen

**Question 1**

**mode** MultipleChoice

**text** Suppose  $A$  and  $B$  are invertible  $n$ -by- $n$  matrices. Which of the following are true:

**correct-choice**  $((AB)^{-1})^T = (A^{-1})^T(B^{-1})^T$

**choice**  $((AB)^{-1})^T = (B^{-1})^T(A^{-1})^T$

**choice**  $((AB)^{-1})^T = B^{-1}A^{-1}B^T A^T$

**choice**  $((AB)^{-1})^T = B^T A^T B^{-1} A^{-1}$

**choice** none of the above and below

**Question 2**

**mode** MultipleChoice

**text** Suppose  $A$  and  $B$  are orthogonal  $n$ -by- $n$  matrices. Which of the following is also an orthogonal matrix?

**choice**  $AB-BA$

**correct-choice**  $\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$

**choice**  $\begin{pmatrix} A & I \\ I & A \end{pmatrix}$

**choice**  $A^2 - B^2$

**choice** none of the above and below

**Question 1**

**mode** MultipleChoice

**text** Let  $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m\}$  be an orthonormal basis of a subspace  $V$  of  $R^n$  and let  $\{\vec{w}_1, \vec{w}_2, \dots, \vec{w}_k\}$  be an orthonormal basis of  $V^\perp$ . Then the union  $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m, \vec{w}_1, \vec{w}_2, \dots, \vec{w}_k\}$  is

**choice** a linearly independent set but not necessarily orthogonal

**choice** an orthonormal set but not necessarily a basis for  $R^n$

**choice** a spanning set for  $R^n$  but not necessarily linearly independent

**correct-choice** an orthonormal basis for  $R^n$

**choice** none of the above and below

**Question 2**

**mode** MultipleChoice

**text** Is an orthogonal projection onto a subspace  $V$  also an orthogonal transformation?

**choice** yes, because they are both orthogonal

**choice** No, because an orthonormal projection is not a square matrix.

**correct-choice** No, unless  $V = R^n$ , because projection does not preserve distance

**choice** No, because an orthogonal projection is not a linear transformation

**choice** Only if  $V$  is a line

**choice** none of the above and below

**Question 1**

**mode** MultipleChoice

**text** Fact 5.4.8 says that  $A(A^T A)^{-1}A^T$  is the matrix of the orthogonal projection onto  $V$ .

But  $(A^T A)^{-1} = A^{-1}(A^T)^{-1}$  so  $A(A^T A)^{-1}A^T = AA^{-1}(A^T)^{-1}A^T = I * I = I$  so the matrix is always identity.

This argument is:

**choice** valid

**choice** invalid because matrix multiplication is not associative

**choice** only valid if  $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$  is an orthonormal basis

**correct-choice** invalid unless  $m = n$  because  $A$  is not a square matrix

**choice** invalid unless  $m = n$  because  $A$  is a square matrix but is not invertible

**Question 2**

**mode** MultipleChoice

**text** At the end of example 4 on p.218-219, the book says: "The sum of the squares of the vertical distances..."

Why vertical distances?

**choice** The vertical distance is the shortest distance to the curve

**choice** The author is too lazy to compute the shortest distance

**correct-choice** the vertical distance is the actual error in using the curve rather than the data point

**choice** using anything other than the vertical distance would be unpatriotic, and subject to the author penalties under the Patriot Act

**choice** why not?

**choice** none of the above and below

**Question 1**

**mode** MultipleChoice

**text** If the entries of a square matrix are positive integers, the determinant could be:

**choice** a positive integer

**choice** a nonnegative integer

**correct-choice** any integer

**choice** any rational number

**choice** any real number

**Question 2**

**mode** MultipleChoice

**text** In definition 6.1.2, some products are added and some products are subtracted.

Let  $A$  and  $S$  denote the number of products that are added and subtracted; For example when  $n = 3$ ,  $A = S = 3$ . Which of the following is true:

**correct-choice**  $A = S$  for all  $n$

**choice**  $A \geq S$  with equality when  $n$  is odd

**choice**  $A \leq S$  with equality when  $n$  is odd

**choice** there is no relationship between  $A$  and  $S$  when  $n$  is large

**choice**  $A = 3$  for all  $n$

**choice**  $S = 3$  for all  $n$

**Question 1**

**mode** MultipleChoice

**text** Let  $A$  be an  $n * n$  matrix, and  $B = kA$  for some constant  $k$ . Then  $\det(B)$  is equal to

**choice**  $k\det(A)$

**correct-choice**  $k^n\det(A)$

**choice**  $\frac{1}{k}\det(A)$

**choice**  $\frac{1}{k^n}\det(A)$

**choice**  $k + \det(A)$

**choice**  $k^n + \det(A)$

**choice** cannot be determined in terms of  $k$  and  $\det(A)$  alone.

**Question 2**

**mode** MultipleChoice

**text** Let  $A$  be an  $n * n$  matrix such that  $\det(A^2) = 1$ . Then

**choice**  $\det(A) = 1$

**choice**  $\det(A) = 0$  or  $1$

**choice**  $\det(A) = -1, 0$  or  $1$

**correct-choice**  $\det(A) = -1$  or  $1$

**choice**  $\det(A)$  may be any positive number

**choice**  $\det(A)$  may be any real number



**Question 1**

**mode** MultipleChoice

**text** Let  $A$  be an  $n * n$  matrix with  $\det(A) = 1$ , and let  $T(\vec{x}) = A\vec{x}$ . Then

**choice**  $T$  must be a rotation

**correct-choice**  $T$  must preserve volume

**choice**  $T$  may be any isomorphism

**choice**  $T$  may be any orthogonal transformation

**choice** none of the above and below

**Question 2**

**mode** MultipleChoice

**text** Let  $A_{ij}$  denote the minors of an  $n * n$  matrix  $A$ . What is the relationship between  $\det(A_{ij})$  and  $\det(A_{ji})$ ?

**choice** They are always equal

**choice**  $\det(A_{ij}) = -\det(A_{ji})$  if  $i \neq j$

**correct-choice** They are equal if  $A$  is a symmetric matrix

**choice** If  $\det(A_{ij}) = 0$  then  $\det(A_{ji}) = 0$

**choice** Their product equals 1.

**choice** none of the above or below

**Question 1**

**mode** MultipleChoice

**text** Suppose  $A$  is an  $n * n$  matrix with integer entries with  $\det(A) = 1$ . What can you say about the entries of  $A^{-1}$ ?

**correct-choice** They must be integers

**choice** They must be rational numbers, but not necessarily integers

**choice** They could be any real numbers

**choice**  $A^{-1}$  might not exist

**choice** The answer depends on whether or not  $n$  is even or odd

**Question 2**

**mode** MultipleChoice

**text** Let  $A$  be an invertible  $n * n$  matrix, with  $\text{adj}(A)$  defined in **Fact 6.3.10**. What is the relationship between  $\det(\text{adj}(A))$  and  $\det(A)$ ?

**choice**  $\det(\text{adj}(A)) = \det(A)$

**choice**  $\det(\text{adj}(A)) = \frac{1}{\det(A)}$

**correct-choice**  $\det(\text{adj}(A)) = (\det A)^{n-1}$

**choice**  $\det(\text{adj}(A)) = \frac{1}{(\det A)^{n-1}}$

**choice**  $\det(\text{adj}(A)) = (\det A)^{n+1}$

**choice** they are not related

**Question 1**

**mode** MultipleChoice

**text** In definition 7.1.1 it says "A nonzero vector  $\vec{v}$  ...". Why is it that  $\vec{v}$  is required to be nonzero?

**choice** Zero is not allowed to be an eigenvalue

**correct-choice** The zero vector would satisfy the equation for any eigenvalue

**choice** The zero vector does not satisfy the equation for any eigenvalue

**choice** All the entries of an eigenvector must be nonzero

**choice** The author dislikes the zero vector

**Question 2**

**mode** MultipleChoice

**text** If  $A$  is a  $2 \times 2$  matrix with eigenvalues 1 and  $-1$ , then what can you say about  $A^2$ ?

**choice**  $A^2$  also has eigenvalues 1 and  $-1$

**correct-choice**  $A^2$  is the identity matrix

**choice**  $A^2$  is the zero matrix

**choice** you can't determine the eigenvalues of  $A^2$

**choice**  $A^2 = A^{-1}$

**Question 1**

**mode** MultipleChoice

**text** What are the eigenvalues of the matrix  $\begin{pmatrix} 2 & 4 \\ 4 & 3 \end{pmatrix}$  ?

**choice** 2 and 3

**choice** 5 and 10

**choice** -5 and -10

**correct-choice**  $\frac{5+\sqrt{65}}{2}$  and  $\frac{5-\sqrt{65}}{2}$

**choice**  $\frac{5+\sqrt{10}}{2}$  and  $\frac{5-\sqrt{10}}{2}$

**choice** the matrix has no real eigenvalues

**Question 2**

**mode** MultipleChoice

**text** What are the roots of  $\det(A - \lambda I_n)$ ?

**choice** The negatives of the eigenvalues of A

**correct-choice** The eigenvalues of A

**choice** The eigenvalues of A when n is even, and the negatives of the eigenvalues of A when n is odd

**choice** Only the eigenvalues of A that are positive

**choice** Only the eigenvalues of A that are negative

**choice** none of the above and below

**Question 1**

**mode** MultipleChoice

**text** Let  $T(\vec{x}) = A\vec{x}$  be the orthogonal projection onto a line  $L$  in  $R^3$ . What can you say about the eigenvalues and eigenspaces of  $T$ ?

**choice** it depends on the line  $L$

**choice** The eigenvalues are  $-1, 0, 1$  with each eigenspace of dimension 1

**choice** The eigenvalues are  $0, 1$  with eigenspaces of dimension 1

**choice** The eigenvalues are  $0, 1$  with corresponding eigenspaces of dimension 1, 2

**correct-choice** The eigenvalues are  $0, 1$  with corresponding eigenspaces of dimension 2, 1

**Question 2**

**mode** MultipleChoice

**text** Suppose  $A$  is an  $n * n$  matrix which has an eigenbasis. To what extent is the eigenbasis unique?

**choice** any other eigenbasis must consist of nonzero multiples of the vectors of a given basis

**correct-choice** if the eigenvalues are distinct, any other eigenbasis must consist of nonzero multiples of the vectors of a given basis

**choice** if all the eigenvalues are different from 0, any other eigenbasis must consist of nonzero multiples of the vectors of a given basis

**choice** if  $A$  is a diagonal matrix, any other eigenbasis must consist of nonzero multiples of the vectors of a given basis

**choice** none of the above or below

**Question 1**

**mode** MultipleChoice

**text** Which of the following conditions on A implies that A has an eigenbasis?

**choice** A is upper triangular

**correct-choice** A is upper triangular with distinct values on the diagonal

**choice** A is upper triangular with nonzero values on the diagonal

**choice** A is upper triangular with all zeros on the diagonal

**choice** A is upper triangular with  $\det(A) = 1$

**choice** none of the above or below

**Question 2**

**mode** MultipleChoice

**text**

**choice** Suppose the  $n \times n$  matrices A and B commute, and  $\vec{v}$  is an eigenvector of A with eigenvalue  $\lambda$ . Which of the following statements is always true?

**choice**  $\vec{v}$  is an eigenvector of B

**choice**  $B\vec{v} = 0$

**correct-choice**  $B\vec{v}$  is an eigenvector of A with eigenvalue  $\lambda$

**choice**  $B\vec{v}$  is an eigenvector of A but its eigenvalue may be any real number

**choice** B must have an eigenbasis

**Question 1**

**mode** MultipleChoice

**text** Suppose A is diagonalizable with  $S^{-1}AS$  diagonal. What can you say about  $A^T$ ?

**choice**  $A^T$  may or may not be diagonalizable

**choice**  $A^T$  is diagonalizable with  $S^{-1}A^T S$  diagonal

**choice**  $A^T$  is diagonalizable with  $(S^T)^{-1}A^T S^T$  diagonal

**correct-choice**  $A^T$  is diagonalizable with  $S^T A^T (S^T)^{-1}$  diagonal

**choice**  $A^T$  is diagonalizable but you can't say anything about the matrix that diagonalizes it

**Question 2**

**mode** MultipleChoice

**text** Suppose A is diagonalizable and  $\lim_{t \rightarrow \infty} A^t$  exists. What can you say about the eigenvalues  $\{\lambda_k\}$  of A?

**choice**  $\lambda_k \geq 0$  for all k

**choice**  $0 \leq \lambda_k \leq 1$  for all k

**choice**  $-1 \leq \lambda_k \leq 1$  for all k

**choice**  $0 < \lambda_k < 1$  for all k

**correct-choice**  $-1 < \lambda_k \leq 1$  for all k

**choice**  $-1 < \lambda_k \leq 1$  for all k and  $\lambda_k = 1$  for some k

**Question 1**

**mode** MultipleChoice

**text** Use the standard basis  $\{1, i\}$  to represent  $C$  as  $R^2$ . Then the operation of multiplication by a fixed complex number is a linear transformation from  $R^2$  to  $R^2$ , so may be represented by a  $2 * 2$  matrix  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ . See example 4 on page 344 for one case. What can you say about the form of this matrix?

**choice** it is symmetric

**choice** it is skew-symmetric

**choice** it is orthogonal

**correct-choice** it satisfies  $a = d$  and  $b = -c$

**choice** it satisfies  $a = -d$  and  $b = c$

**choice** it may be any  $2 * 2$  matrix

**Question 2**

**mode** MultipleChoice

**text** In DeMoirre's formula (p.345) the angle  $\phi$  is only determined up to a multiple of  $2\pi$ . How do different choices of  $\phi$  affect this formula?

**correct-choice** It doesn't matter because the sine and cosine are also periodic

**choice** The formula requires  $0 \leq \phi < 2\pi$

**choice** The formula requires  $-\pi < \phi \leq \pi$

**choice** There are in general  $n$  different  $n^{th}$  powers of a complex number

**choice** When  $n$  is odd, the  $n^{th}$  power is only determined up to a sign (plus or minus).



**Question 1**

**mode** MultipleChoice

**text** Suppose  $A$  is an  $n \times n$  (real) matrix, with  $m$  distinct real eigenvalues and  $k$  distinct complex eigenvalues. What are the allowed values of  $m$  and  $k$ ?

**choice**  $m + k = n$ , and  $m$  is even

**choice**  $m + k = n$ , and  $k$  is even

**choice**  $m + k \leq n$ , and  $m$  is even

**correct-choice**  $m + k \leq n$ , and  $k$  is even

**choice**  $m + 2k \leq n$

**Question 2**

**mode** MultipleChoice

**text** Suppose  $A$  is a  $3 \times 3$  matrix with complex eigenvalues  $a + ib, a - ib$  (for  $b \neq 0$ ) and real eigenvalue  $\lambda$ . What condition implies  $\det(A) > 0$ ?

**choice**  $a > 0$

**choice**  $b > 0$

**correct-choice**  $\lambda > 0$

**choice**  $\det(A)$  is always positive

**choice**  $\det(A)$  is never positive

**choice** You can't tell from eigenvalues alone

**Question 1**

**mode** MultipleChoice

**text** Suppose the initial values are close to zero. Under what conditions on the matrix  $A$  will the solution to the linear system  $\frac{d\vec{x}}{dt} = A\vec{x}$  stay close to zero for all positive times?

**choice**  $A$  is diagonalizable

**choice**  $\det(A) < 0$

**choice**  $A$  is diagonalizable and all eigenvalues are  $> 0$

**correct-choice**  $A$  is diagonalizable and all eigenvalues are  $< 0$

**choice** no condition on  $A$  will be sufficient

**Question 2**

**mode** MultipleChoice

**text** For a continuous dynamical system, we are interested in whether an eigenvalue is  $> 0$  or  $< 0$ . For a discrete dynamical system, the distinction was whether  $\lambda > 1$  or  $0 < \lambda < 1$  (we didn't even discuss the interpretation of negative eigenvalues). How are these questions related?

**correct-choice**  $e^\lambda > 1$  if and only if  $\lambda > 0$

**choice**  $0 < 1$

**choice** continuous time is exact, whereas discrete time is only approximate

**choice** these questions are unrelated

**Question 1**

**mode** MultipleChoice

**text** Suppose  $\frac{d\vec{x}}{dt} = A\vec{x}$  has a periodic orbit. What does this imply about the matrix  $A$ ?

**choice** All the eigenvalues of  $A$  are real

**choice** All the eigenvalues of  $A$  are complex

**correct-choice** Some of the eigenvalues of  $A$  are purely imaginary

**choice** All of the eigenvalues of  $A$  are purely imaginary

**choice**  $A$  is diagonalizable

**Question 2**

**mode** MultipleChoice

**text** Suppose  $A$  is a  $2 \times 2$  matrix, and one solution of  $\frac{d\vec{x}}{dt} = A\vec{x}$  is an outward spiral. What can you say about all nonconstant solutions?

**choice** Some are outward spirals and some are inward spirals

**correct-choice** All are outward spirals

**choice** Some will spiral clockwise and others will spiral counterclockwise

**choice** Not all will be spirals

**choice** You can't predict, since the answer depends on  $A$

**Question 1**

**mode** MultipleChoice

**text** In exaple 5 on p.429, all solutions of the equation  $f'' + 2f' - 3f = 0$  are formed to be of the form  $c_1e^t + c_2e^{-3t}$ . Suppose we want a solution with  $f(0) = a$  and  $f'(0) = b$  for specified constants a and b.

**choice** take  $c_1 = a$  and  $c_2 = b$

**choice** take  $c_1 = \frac{1}{4}(a - b)$  and  $c_2 = \frac{1}{4}(3a + b)$

**correct-choice** take  $c_1 = \frac{1}{4}(3a + b)$  and  $c_2 = \frac{1}{4}(a - b)$

**choice** take  $c_1 = 3a$  and  $c_2 = -b$

**choice** it is not possible to find a solution in general

**Question 2**

**mode** MultipleChoice

**text** In fact 9.3.13 there is an indefinite integral. Such integrals are only defined up to an additive constant. Does it matter which constant we choose?

**choice** Any choice of the constant leads to the same solution

**choice** Different choices of the constant lead to solutions of the equation which differ by a constant

**correct-choice** Different choices of the constant lead to different solutions of the inhomogeneous equation which differ by a multiple of  $e^{at}$  (a solution of the homogeneous equation)

**choice** You have to choose zero for the constant

**choice** There is only one allowable choice for the constant, and it depends on  $a$

**Question 1**

**mode** MultipleChoice

**text** Is a subspace of an inner product space also an inner product space (using the same inner product)?

**correct-choice** Yes

**choice** Only when it is an orthogonal subspace

**choice** Only when it is the whole space

**choice** Only when you can verify condition (d) of Definition 5.5.1

**Question 2**

**mode** MultipleChoice

**text** Consider the inner product in Exaple 1 on page 226. Suppose  $f$  is orthogonal to  $g$ , and  $g$  is a positive function. What can you say about  $f$ ?

**choice**  $f$  is a positive function

**choice**  $f$  is a nonnegative function

**choice**  $f$  is a negative function

**choice**  $f$  is never zero

**correct-choice**  $f$  has at least one zero

**choice**  $f$  has exactly one zero

**Question 1**

**mode** MultipleChoice

**text** Let  $f_n$  and  $f_m$  be the Fourier approximations of  $f$  defined in *Fact5.5.5* with  $n < m$ . What can you say about the same coefficients  $b_k, c_k$  and  $a_0$  for the two approximations?

**choice** they are the same

**correct-choice** they are the same for  $k \leq n$

**choice** they are the same for  $k \leq n$  and  $b_k = c_k = 0$  for  $k > n$

**choice**  $a_0$  is the same, but  $b_k$  and  $c_k$  are unrelated

**choice** they are all unrelated

**Question 2**

**mode** MultipleChoice

**text** Suppose we know for a particular value of  $n$  that the error  $\|f - f_n\| \leq \frac{1}{1000}$ , where  $f_n$  is the Fourier approximation of  $f$ . What can be said about  $|f(t) - f_n(t)|$  for a particular value of  $t$ ?

**choice**  $|f(t) - f_n(t)| \leq \frac{1}{1000}$  for all  $t$

**choice**  $|f(t) - f_m(t)| \leq \frac{1}{1000}$  for all  $t$  if  $m$  is large enough

**correct-choice**  $|f(t) - f_n(t)| \leq \frac{1}{1000}$  on average, but nothing can be said for a specific value of  $t$ .

**choice**  $|f(t) - f_n(t)| \leq \frac{1}{1000}$  except for a finite value number of  $t$  values

**choice** it is possible that  $|f(t) - f_n(t)| > \frac{1}{1000}$  for all  $t$