1. Find all real eigenvalues of the matrix

\[ A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix}. \]

The matrix is in block diagonal form with blocks \( B = (4) \) and \( C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}. \)

To find the eigenvalues of \( C \), compute

\[ \det \begin{pmatrix} 1 - t & 1 \\ 2 & 2 - t \end{pmatrix} = (1 - t)(2 - t) - 2 = t^2 - 3t = t(t - 3). \]

This tells us that the eigenvalues of \( C \) are 0 and 3, so the eigenvalues of \( A \) are 0, 3 and 4.