1. Find the intervals on which the function is increasing, decreasing, concave up and concave down.

(a) $f(x) = 5 + 3x - x^2$
(b) $f(x) = 2x^3 - 9x^2 + 12x + 1$
(c) $f(x) = \frac{1}{x}$
(d) $f(x) = 3x - 1$
(e) $f(x) = \tan^{-1} x$
(f) $f(x) = \sin x$
(g) $f(x) = e^x - e^{2x}$

2. How many real solutions are there to the equation $x^7 + x^5 + x^3 + x = 100$? How do you know?

3. (4.2 #35) Two runners start a race at the same time and finish in a tie. Prove that at some time during the race they have the same speed. (Hint: Consider $f(t) = g(t) - h(t)$, where $g$ and $h$ are the position functions of the two runners.)

4. (a) Use the linear approximation to the function $f(x) = \sin^{-1} x$ at $a = 0$ to “estimate” $f\left(\frac{1}{2}\right)$. But we already know the value of $\sin^{-1} \frac{1}{2}$. (What is it?) Plug it in to get an approximate equation involving $\pi$. Then solve for $\pi$ to find an approximate value for $\pi$.

(b) To get a more accurate approximation to $\pi$, use the half-angle formula

$$\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}}$$

to find $\sin \frac{\pi}{12}$. Then use the linear approximation at $a = 0$ to “estimate” $\sin^{-1}(\sin \frac{\pi}{12})$. (People use this kind of idea to compute $\pi$ out to thousands of decimal places.)