1. Let $X_t$ be a continuous time Markov chain with states $0, 1, 2, 3$ and jump rates

\[ q(0, 1) = 5, \quad q(0, 2) = q(0, 3) = 3, \quad q(1, 2) = q(1, 3) = q(2, 3) = 1, \quad q(3, 2) = 4. \]

(a) Draw a picture. Which states are recurrent?

(b) Find $P_0(T_1 < \infty)$ and $P_0(T_2 < \infty)$.

(c) Starting in state 0 what is the total expected amount of time spent in state 1?

(d) Starting in state 2 what is the probability of being in state 3 at time $t = 10$?

(e) Starting in state 0 what is the long term fraction of time we spend in state 3?

2. (a) Uranium has a half-life of $4.5 \times 10^9$ years. This means the time $T$ it takes a uranium atom to decay into a lead atom is exponentially distributed with some rate $\lambda$, such that $P(T > 4.5 \times 10^9) = \frac{1}{2}$. Find $\lambda$.

(b) A gram of uranium contains $X_0 = 2.5 \times 10^{21}$ atoms. Each atom decays into lead independently at rate $\lambda$. Let $X_t$ be the number of uranium atoms remaining after $t$ years. Convince yourself that $X_t$ is a continuous time Markov chain. Find its jump rates.