Math 1A Worksheet 16
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12/2/04

1. Find the integral.
   (a) $\int_0^2 |2x-1|dx$
   (b) $\int_{-1}^1 (1-\sqrt{1-x^2})dx$
   (c) $\int_1^4 \sqrt{8+2x-x^2}dx$
   (d) $\int_0^{7\pi/2} |\sin x|dx$
2. If $\int_0^x f(t)dt = e^x + x - 1$, what is $f(t)$?
3. If $\int_2^{x^2} f(t)dt = x^6 - 8$, what is $f(t)$?
4. (a) Let $h_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n}$. For example, $h_2 = 1 + \frac{1}{2} = \frac{3}{2}$, and $h_3 = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$. Prove that
   $$h_n > \ln n.$$
   (Hint: express h_n as an approximation to the integral $\int_1^n \frac{1}{x}dx$.)
   (b) What can you conclude about $\lim_{n\to\infty} h_n$?
   (c) Now let $k_n = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots + \frac{1}{n^2}$. Prove that
   $$k_n < 2 - \frac{1}{n}.$$
   (d) What can you conclude about $\lim_{n\to\infty} k_n$?