

Math 413, Spring 2006 — Week 6

EP6-1 Suppose D is a non-empty closed subset of \mathbb{R} . Let

$$\text{dist}(x, D) = \inf\{|x - y| : y \in D\}.$$

- (a) Show that for each $x \in \mathbb{R}$ there is a $y \in D$ with $\text{dist}(x, D) = |x - y|$.
- (b) Show that $f(x) = \text{dist}(x, D)$ is a continuous function.
- (c) Show that $D = \{x \in \mathbb{R} : f(x) = 0\}$.