Hand in solutions to the problems below. due April 7. You can email your problem set to the grader, and Cc the lecturer. You can also hand in your pset in class.

**P1** Prove that if the largest antichain of a finite poset $P$ has $k$ elements, then the set underlying $P$ is the union of the sets underlying $k$ chains of $P$.

**P2** Calculate the Möbius function values $\mu(I, J)$ of any distributive lattice $J(P)$. (give the conditions in terms of the order ideals $I$ and $J$ in $P$.)

**P3** Show that the number of permutations $\pi$ of $[n]$ such that $\pi(i) \neq i$ for $i \in [n]$ is approximately $\frac{n!}{e}$.

**P4** Let $\psi(n)$ be the number of numbers in $[n]$ which are relatively prime to $n$. Express $\psi(n)$ in terms of $n$ and the prime factors of $n$.

**P5** (optional extra credit) Let $des(w)$ be the number of descent of permutation $w$. Let $A(n, k)$ be the number of permutations of $[n]$ with $k - 1$ descents. Let $A_n(x) = \sum_{k=1}^{n} A(n, k)x^k$. Prove that:

(a) $A(n, k) = A(n, n + 1 - k)$

(b) $A(n, k) = kA(n - 1, k) + (n + 1 - k)A(n - 1, k - 1)$

(c) $\frac{A_n(x)}{(1-x)^{n+1}} = \sum_{k=0}^{\infty} k^n x^k$

**P6** (optional extra credit) Interpret the determinant we obtained for the number of permutations of $[n]$ with a given set of descents in terms of lattice paths (number of routings). Prove bijectively (without using the determinant) that the two quantities are equal.