MATH 4550: Applicable Geometry

Problem Set 1

Due at 2:54pm before class starts on January 31, 2013

You are allowed to work in groups, but the solutions you hand in should be written by you only. If you work in a group, you must write the names of your collaborators at the top of your assignment. Explain your reasoning to receive full credit. All problems are worth 10 points. You are strongly encouraged to type your solutions in LaTeX. In any case, please staple your psets!

P1 (a) Prove that the intersection of convex sets is convex.

(b) Recall that we defined the convex hull of $p_1, \ldots, p_t$ as the set of all the convex combinations of $p_1, \ldots, p_t$ and denoted it by $\text{conv}(\{p_1, \ldots, p_t\})$. Prove that:

- $\text{conv}(\{p_1, \ldots, p_t\})$ is convex
- $\text{conv}(\{p_1, \ldots, p_t\})$ is the intersection of all convex sets containing $p_1, \ldots, p_t$

P2 Check that the $d$-cube $C_d$ is the common intersection of $2d$ halfspaces in $\mathbb{R}^d$.

P3 Prove that the intersection of a set of affine subspaces of $\mathbb{R}^d$ is an affine subspace of $\mathbb{R}^d$.

P4 We say that vectors $x_1, \ldots, x_n$ are **affinely independent** if their affine hull (also called affine span) has dimension $n - 1$. Let

$$
\begin{bmatrix}
  x_{1,1} \\
  \vdots \\
  x_{d,1}
\end{bmatrix}, \ldots, 
\begin{bmatrix}
  x_{1,n} \\
  \vdots \\
  x_{d,n}
\end{bmatrix}
$$

1-1
be $n$ elements of $\mathbb{R}^d$ written as column vectors. Prove that $x_1, \ldots, x_n$ are affinely independent if and only if

$$
\hat{x}_1 = \begin{bmatrix} 1 \\ x_{1,1} \\ \vdots \\ x_{d,1} \end{bmatrix}, \ldots, \hat{x}_n = \begin{bmatrix} 1 \\ x_{1,n} \\ \vdots \\ x_{d,n} \end{bmatrix}
$$

are linearly independent in $\mathbb{R}^{d+1}$.

\[ ...... \]

**P5* (extra credit)** Prove that there exists a 3-polytope with $v$ vertices, $e$ edges and $f$ 2-dimensional faces if and only if the following conditions hold:

$$
v - e + f = 2, \ v \leq 2f - 4, \text{ and } f \leq 2v - 4.
$$

You can assume $v - e + f = 2$. 

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