Glossary of Notation

This document is intended as a glossary, or dictionary, of notation for MATH 1110. It will be periodically updated throughout the semester to reflect material covered.

1 Sets

Given sets $A$ and $B$:

$x \in A$ means that $x$ is an element of $A$.

$x \notin A$ means that $x$ is not an element of $A$.

$A \subseteq B$ means that $A$ is a subset of $B$, i.e., every element of $A$ is also an element of $B$.

$\{x : \text{some property holds of } x\} = \text{the set of all elements } x \text{ for which the property described holds.}$

This is also often written as $\{x \mid \text{some property holds of } x\}$, or $\{x \in A : \text{some property holds of } x\}$ if we want to emphasize that the elements of this set are to be taken from $A$.

$A \cup B = \{x : x \in A \text{ or } x \in B \text{ (or both)}\} = \text{the union of } A \text{ and } B$, i.e., the set of all elements which are either in $A$ or in $B$ (or both).

$A \cap B = \{x : x \in A \text{ and } x \in B\} = \text{the intersection of } A \text{ and } B$, i.e., the set of all elements which are either in both $A$ and $B$.

$A \setminus B = \{x : x \in A \text{ and } x \notin B\} = \text{the set which consists of all elements of } A \text{ which are not in } B$.

$A \times B = \{(x, y) : x \in A \text{ and } y \in B\} = \text{the cartesian product of the sets } A \text{ and } B$.

2 The Real Line

$\mathbb{R}$ (a “blackboard bold” letter ‘R’) = the set of real numbers.

$\mathbb{N} = \{0, 1, 2, 3, \ldots\} = \text{the set of natural numbers}$.

$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \ldots, \} = \text{the set of integers}$.

$\mathbb{Q} = \{\frac{m}{n} : m, n \in \mathbb{Z} \text{ and } n \neq 0\} = \text{the set of rational numbers}$.

The following table lists all of the possible intervals in $\mathbb{R}$. For $a < b$ in $\mathbb{R}$:

<table>
<thead>
<tr>
<th>Interval notation</th>
<th>Set description</th>
<th>Terminology</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(a, b)$</td>
<td>${x : a &lt; x &lt; b}$</td>
<td>open</td>
</tr>
<tr>
<td>$[a, b)$</td>
<td>${x : a \leq x &lt; b}$</td>
<td>half-open</td>
</tr>
<tr>
<td>$(a, b]$</td>
<td>${x : a &lt; x \leq b}$</td>
<td>half-open</td>
</tr>
<tr>
<td>$[a, b]$</td>
<td>${x : a \leq x \leq b}$</td>
<td>closed (and bounded)</td>
</tr>
<tr>
<td>$(a, \infty)$</td>
<td>${x : x &gt; a}$</td>
<td>open</td>
</tr>
<tr>
<td>$[a, \infty)$</td>
<td>${x : x \geq a}$</td>
<td>closed (and unbounded)</td>
</tr>
<tr>
<td>$(-\infty, b)$</td>
<td>${x : x &lt; b}$</td>
<td>open</td>
</tr>
<tr>
<td>$(-\infty, b]$</td>
<td>${x : x \leq b}$</td>
<td>closed (and unbounded)</td>
</tr>
</tbody>
</table>
3 Functions

For a function \( f \):

\[
\text{dom}(f) = \text{domain}(f) = \text{the domain of } f.
\]
\[
\text{ran}(f) = \text{range}(f) = \{ f(x) : x \in \text{dom}(f) \} = \text{the range of } f.
\]
\[
\text{graph}(f) = \{(x, f(x)) : x \in \text{dom}(f)\} = \text{the graph of } f.
\]

A \textit{piecewise function} is one defined by a formula such as

\[
f(x) = \begin{cases} 
  \text{(some value depending on } x\text{),} & \text{if } x \text{ satisfies some property} \\
  \text{(some other value depending on } x\text{),} & \text{if } x \text{ satisfies some other property.}
\end{cases}
\]

(There can more than two clauses.)

For functions \( f \) and \( g \), the \textit{composition} of \( f \) and \( g \), denoted \( f \circ g \), is defined by

\[
(f \circ g)(x) = f(g(x)).
\]

This has domain \( \{x : x \in \text{dom}(g) \text{ and } g(x) \in \text{dom}(f)\}\).

The \textit{exponential function} with base \( a \), for \( a > 0 \) and \( a \neq 1 \), is given by \( y = a^x \). This has domain all of \( \mathbb{R} \).

The \textit{natural exponential function} is given by \( y = e^x \), where \( e = 2.7182818\ldots \) is Euler’s number. This has domain all of \( \mathbb{R} \).

The \textit{logarithm function} with base \( a \), for \( a > 0 \) and \( a \neq 1 \), is given by \( y = \log_a(x) \). This has domain \( (0, \infty) \).

The \textit{natural logarithm function} is given by \( y = \ln(e) = \log_e(x) \). This has domain \( (0, \infty) \).

For \( f \) a one-to-one function, the \textit{inverse} of \( f \) is \( f^{-1} \). That has domain \( \text{ran}(f) \).

4 Limits

The \textit{average rate of change} of \( y = f(x) \), with respect to \( x \), on the interval \([x_1, x_2]\) is

\[
\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.
\]

For a function \( y = f(x) \) defined on an open interval containing \( a \), except possibly at \( a \), the \textit{limit} of \( f(x) \) \textit{as } \( x \text{ approaches } a \), if it exists, is denoted by

\[
\lim_{x \to a} f(x).
\]

For a function \( y = f(x) \) defined on an open interval \((a, c)\), for some \( c > a \), the \textit{limit} of \( f(x) \) \textit{as } \( x \text{ approaches } a \text{ from the right} \), if it exists, is denoted by

\[
\lim_{x \to a^+} f(x)
\]

For a function \( y = f(x) \) defined on an open interval \((c, a)\), for some \( c < a \), the \textit{limit} of \( f(x) \) \textit{as } \( x \text{ approaches } a \text{ from the left} \), if it exists, is denoted by

\[
\lim_{x \to a^-} f(x)
\]