Additions to Second Edition Errata and Comments

June 17, 2002

We again thank Dick Palas for his contributions.

Page 271 Last line: "an inverse function," not "a inverse function."

Page 332 Line immediately before Equation 3.3.38: "$(-\frac{1}{3}!)h_1^3$" should be "$(-\frac{1}{3!})h_1^3$."

Page 569 The last margin note refers to nonexistent parts a) and b) of Definition 6.1.1. That sentence should read

The wedge product $\varphi \wedge \omega$ satisfies the requirements of Definition 6.1.1 for a form (multilinearity and antisymmetry).

Page 584 In Equation 6.3.9, the second equality is incorrect; the second determinant is opposite the first. The discussion should read:

... so we are looking for either

$$\omega_{\mathbf{x}}(\vec{\mathbf{v}},\vec{\mathbf{w}}) = \det\begin{bmatrix} y & 0 & v_1 & w_1 \\ x & 2y & v_2 & w_2 \\ w & 2z & v_3 & w_3 \\ z & 0 & v_4 & w_4 \end{bmatrix} \quad \text{or} \quad \omega'_{\mathbf{x}}(\vec{\mathbf{v}},\vec{\mathbf{w}}) = \det\begin{bmatrix} 0 & y & v_1 & w_1 \\ 2y & x & v_2 & w_2 \\ 2z & w & v_3 & w_3 \\ 0 & z & v_4 & w_4 \end{bmatrix}. \tag{6.3.9}$$

These 2-forms are nonzero elements of $A^2(T_{\mathbf{x}}S)$, i.e., $\omega_{\mathbf{x}}(\vec{\mathbf{v}},\vec{\mathbf{w}}) = -\omega'_{\mathbf{x}}(\vec{\mathbf{v}},\vec{\mathbf{w}}) \neq 0$ if $\vec{\mathbf{v}},\vec{\mathbf{w}} \in T_{\mathbf{x}}S$ are linearly independent. The first gives

$$\begin{aligned}\omega_{\mathbf{x}} = -2z^2\,dx\wedge dy + 2yz\,dx\wedge dz + (2xz-2yw)\,dx\wedge dw \\ + 2y^2\,dz\wedge dw - 2zy\,dy\wedge dw.\end{aligned} \tag{6.3.10}$$

Page 591 Third margin note: Definition 6.4.2, not 6.4.1.

Page 595 Margin note half-way down the page: Equation 6.4.20, not 6.4.19.

Page 609 Last margin note: the signs are reversed in the matrix; it should be $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

Page 631 In the first line of Equation 6.7.14, φ should be ψ.

Page 632 In Theorem 6.7.7, we should have said, "For any k-form φ of class C^2 ... ."

Page 633 Margin note: Theorem 6.7.8, not Theorem A6.7.8.

Page 634 Exercise 6.7.6: "Compute the following exterior derivatives," not "Compute the exterior following derivatives."

Exercise 6.7.7: In part (b), "check the computation in (b)" should be "check the computation in (a)."

Page 640 In the margin note, curl $\vec{F}$ should be curl curl $\vec{F}$. In $\mathbb{R}^3$ the Laplacian is often denoted Δ. Note that Δ is the dot product $\nabla \cdot \nabla$:

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} \cdot \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = D_1^2 + D_2^2 + D_3^2.$$

Thus Δ is sometimes denoted ∇^2.

Page 642 Part (c) of Exercise 6.8.11 was not clearly stated. We mean that you should compute them directly from the definition of the exterior derivative. We strongly recommend doing at least part of part (c).

Page 659 We should have chosen our bicycle trip at the top of the hill; then it would be clear that if a cyclist starts and ends at the same point, he or she does no work against gravity. In the absence of friction (including friction from braking) a cyclist could zoom down one hill and coast back up the next, without doing any work.

Page 661 In the margin note, the reference should be to Equation 6.5.12, not 5.6.1.

Page 662 The function described in Theorem 6.11.5 is unique up to the addition of an arbitrary constant. Thus the function given in Equation 6.11.24 is not the only potential of the vector field; any function $\dfrac{xy^2}{2} + xyz + c$, where c is an arbitrary constant, is also a potential of $\vec{F}$.

Page 664 Exercise 6.1.3, part (b): "Sketch the potential" should be "sketch the electric field."

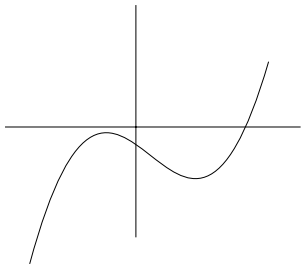
Page 666 Exercise 6.12: the matrix should be $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. This affects parts (a) and (b).

Page 667 In Exercise 6.18, part (b), the displayed equation should be

$$\mathrm{vol}_n\big(B_1^n(\mathbf{0})\big) = \frac{1}{n}\,\mathrm{vol}_{n-1}(S^{n-1}).$$

Page 675 Proposition A2.4: By "exactly" we mean "if and only if." In any case, "if and only if" is more appropriate here. We tend to use "precisely" (or, more rarely, "exactly") when we mean "if and only if" but where the result is fairly obvious, which isn't the case here.

The bottom graph in Figure A2.1 is wrong; it should be:



Page 765 The margin note should start with "In," not "in."

Page 767 In the last margin note, an end parenthesis is missing: $g(P_{\mathbf{f(x)}}$ should be $g(P_{\mathbf{f(x)}})$

Page 770 We have rewritten the first paragraph:

Why does this result matter? To define the exterior derivative, we used the parallelograms $P_{\mathbf{x}}(\vec{\mathbf{v}}_1, \ldots, \vec{\mathbf{v}}_k)$. To do this, we had to know how to draw straight lines from one point to another; we were using the linear (straight) structure of a vector space. (We used $\mathbb{R}^n$, but any vector space would have done.) Theorem A24.8 says that "curved parallelograms" (little bits of manifolds) would have worked as well. Thus the exterior derivative is not restricted to forms defined on vector spaces.

(In this book we have discussed forms on vector spaces, but differential forms can also be defined on manifolds embedded in $\mathbb{R}^n$ and on abstract manifolds. Theorem A24.8 says that an exterior derivative exists for such forms. It is a crucial result, since forms without an exterior derivative would be of no interest.)

Title of Theorem A24.8: By "intrinsic" we mean "inherent: independent of some external conditions or circumstances." The pullback of a form by a C^1 mapping is a C^1 change of variables. Equation A24.17 says that when a form is pulled back by a C^1 mapping, its exterior derivative remains the same, translated appropriately into the new variables.

Equation A24.19: above the first equal sign, "Theorem A6.7.8" should be "Theorem 6.7.8."

Additions to index: